



25-28 April 2017
Aalborg

Knowledge FOR Resilient soCiEty

**Systems Engineering -
Decision Analysis in Engineering**

*Michael Havbro Faber Aalborg
University, Denmark*

The European Commission support for the production of this publication does not constitute an endorsement of the contents which reflects the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained



AALBORG UNIVERSITY
DENMARK

Co-funded by the
Erasmus+ Programme
of the European Union



Contents of Presentation

- **Context of Engineering Decision Making**
- **Decisions and preferences**
- **Uncertainty**
- **Probability**
- **Decision Ranking**
- **Introduction to Decision Analysis**

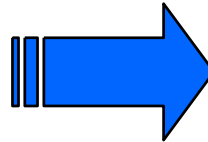
Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



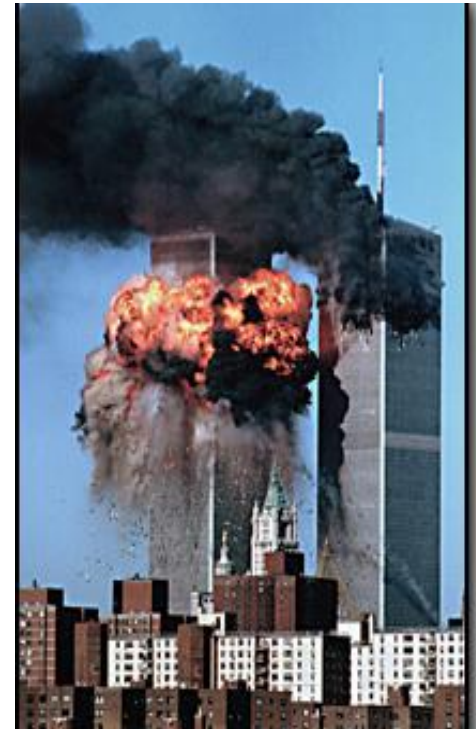
Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Deepwater Horizon

April 20, 2010

11 fatalities

17 injured

Oil spill > 5 million barrels

Health effects?

Eco. imp. > 10 billion \$US

BP response – 14 billion \$US

22000 lost jobs

Context of Engineering Decision Making

What are we up against?



Hurricane Katrina

August 23, 2005

> 1800 fatalities

Eco. imp. > 80 billion \$US

Context of Engineering Decision Making

What are we up against?



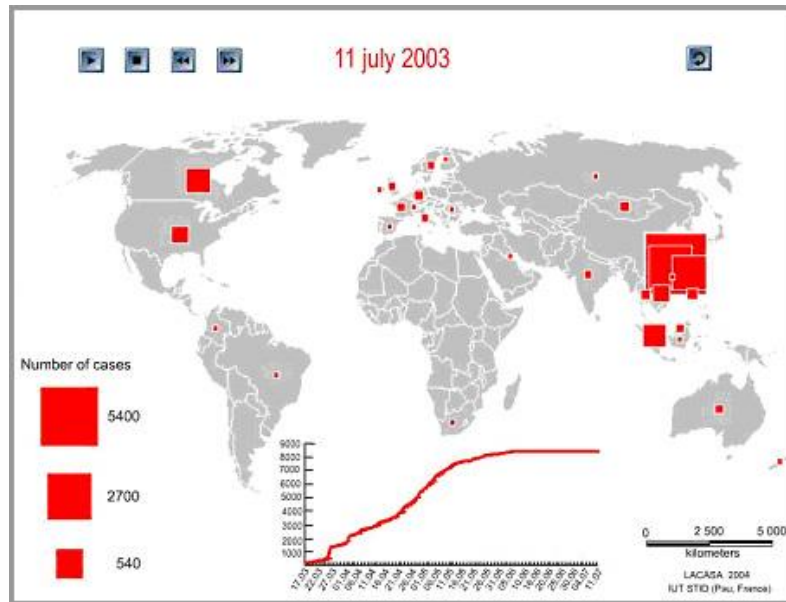
Fukushima Nuclear Event
March 11, 2011

No fatalities ..?

Eco. imp. > 75 billion \$US

Context of Engineering Decision Making

What are we up against?



SARS, 2003

Fatalities: < 800

Eco. imp. 2% GDP – 200 billion \$US

Context of Engineering Decision Making

What are we up against?



Food borne diseases - USA

Affects 76 million per year

Hospitalizations: 325000 per year

Fatalities: 5000 pr year

Decisions and Preferences

Attributes of decision outcomes

Decisions aim to achieve an objective

The degree of achievement is measured by attributes

- natural attributes (measurable, e.g. costs and loss of lives)
- constructed attributes (a function of natural attributes e.g. GDP)
- proxy attributes (indicators which measure the perceived degree of fulfilment of an objective)

Decisions and Preferences

Preferences among attributes - utility

The attributes associated with a decision outcome may be translated into a degree of achievement of the objective by means of a utility function

different attributes are brought together on one or several scales

multi attribute decision making implies a weighing of different attributes

Decisions and Preferences

Constraints on decision making

In principle – any society may define what they consider to be acceptable decisions

Typically decisions are constrained – e.g. in terms of maximum acceptable risks to

- persons
- qualities of the environment

Uncertainty

Different types of uncertainties influence decision making

Inherent natural variability – aleatory uncertainty

- result of throwing dices
- variations in material properties
- variations of wind loads
- variations in rain fall

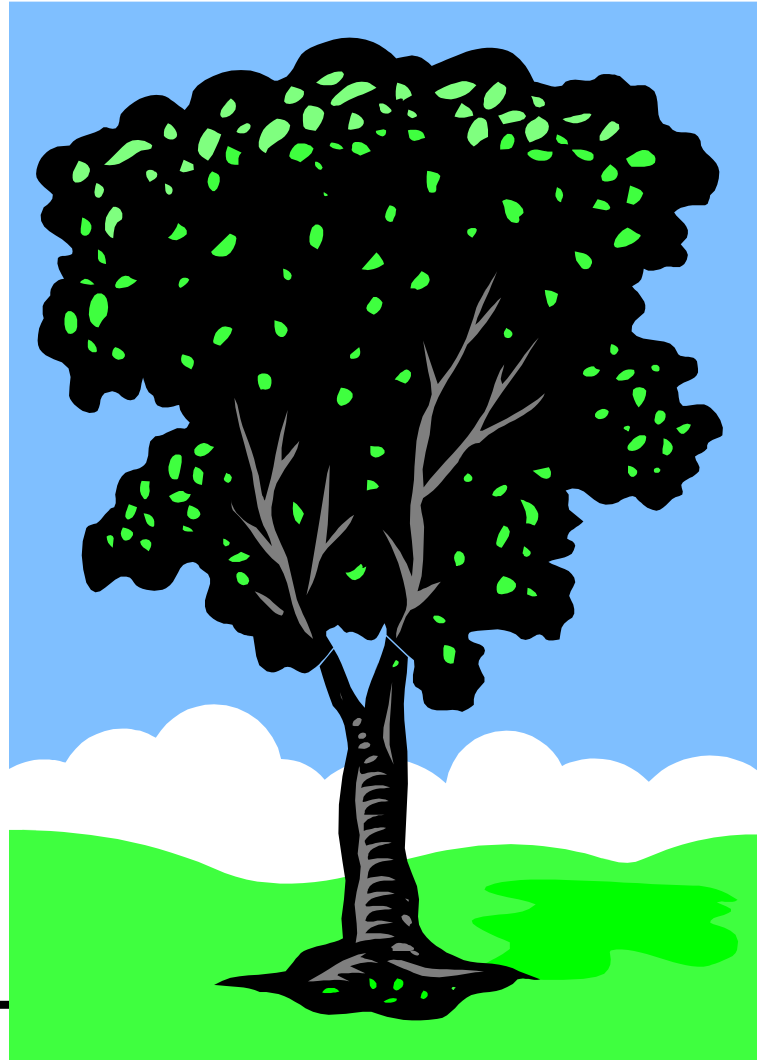
Model uncertainty – epistemic uncertainty

- lack of knowledge (future developments)
- inadequate/imprecise models (simplistic physical modelling)

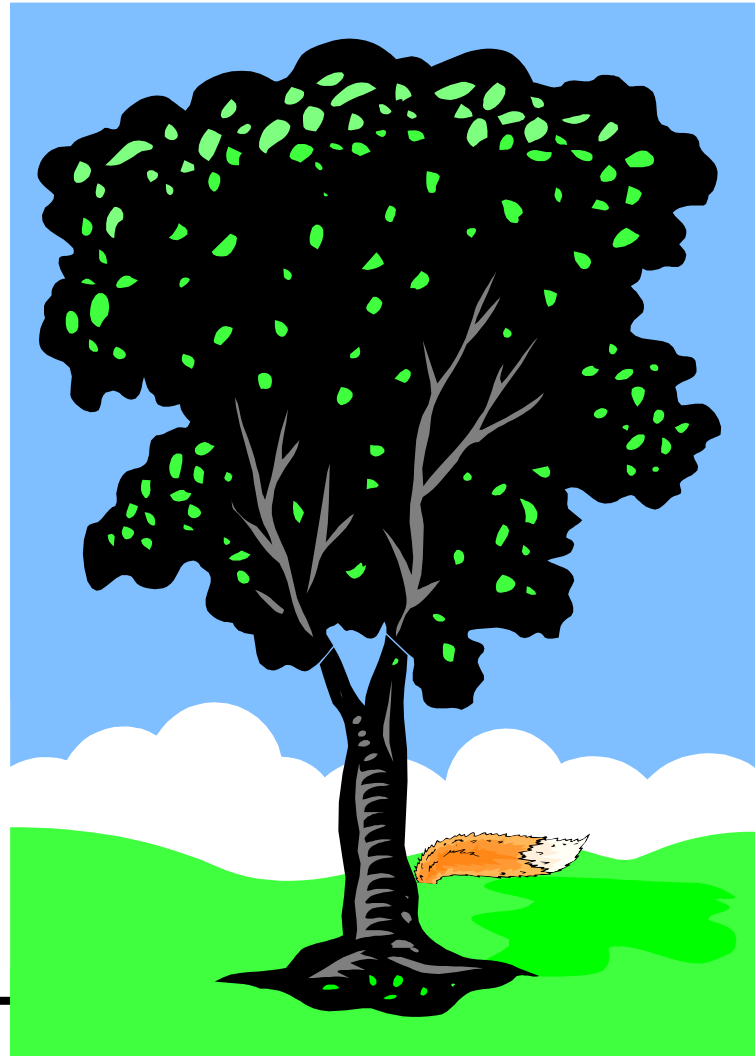
Statistical uncertainties – epistemic uncertainty

- sparse information/small number of data

Probability



Probability



Probability



Probability

Formulate hypothesis about the world



Utilize existing knowledge



Combine with data



Learn how to develop knowledge !

Probability

Conditional probabilities are of special interest as they provide the basis for utilizing new information in decision making.

The conditional probability of an event E_1 given that event E_2 has occurred is written as:

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \quad \text{Not defined if } P(E_2) = 0$$

The event E_1 is said to be probabilistically independent of the event E_2 if:

$$P(E_1|E_2) = P(E_1)$$

Probability

From $P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

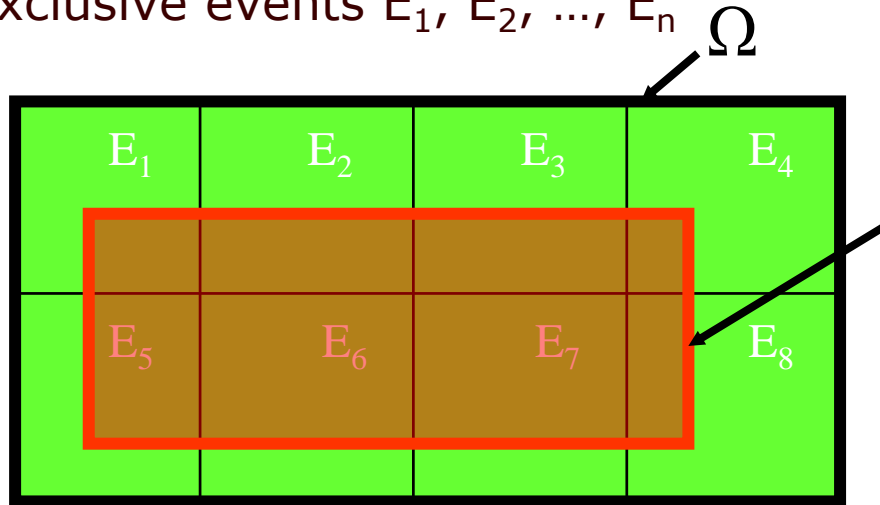
it follows that $P(E_1 \cap E_2) = P(E_2)P(E_1|E_2)$

and when E_1 and E_2 are statistically independent it is

$$P(E_1 \cap E_2) = P(E_2)P(E_1)$$

Probability

Consider the sample space Ω divided up into n mutually exclusive events E_1, E_2, \dots, E_n



$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$$

$$P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \dots + P(A|E_n)P(E_n) =$$

$$\sum_{i=1}^n P(A|E_i)P(E_i)$$

Probability

as there is $P(A \cap E_i) = P(A|E_i)P(E_i) = P(E_i|A)P(A)$

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^n P(A|E_i)P(E_i)}$$

Posterior

Bayes' Rule



Decision Ranking

Emperor Qianlong

Qing dynasty

Reign :1735 – 1796

Daniel Bernoulli 1738

Expected utility hypothesis

von Neumann and Morgenstern 1947

4 Axioms of utility theory:

Ranking based on expected value
of utility (VNM rational)



ZHONG HE DIAN (Hall of Central Harmony)

First constructed in 1420 during the Ming Dynasty, Zhong He Dian was destroyed and reconstructed several times over the centuries. The existing hall was constructed in 1627 during The Ming Dynasty. In the early Ming Dynasty, this hall was called Hua Gai Dian (Hall of Overwhelming Glory), but was renamed Zhong Ji Dian (Hall of Central Extremity) in 1562 and Zhong He Dian in 1645 during the Qing Dynasty. This square building has a single pyramid-shaped roof, with a gold plated bronze covering. The floor is paved with high-quality square clay bricks, commonly known as "golden bricks." A throne is placed in the center of the hall and a board hangs above the throne with an inscription written by Emperor Qianlong. The inscription reads: "Yun Zhi Jue Zhong," meaning "The Way of Heaven is profound and mysterious and the way of mankind is difficult. Only if we make a precise and unified plan and follow the doctrine of the mean, can we rule the country well."

This hall served as a resting place for the emperor on his way to attend an important ceremony or hold court. Officials kowtowed to the emperor here. The day before the emperor held a sacrificial ceremony he would read the prayer tablet aloud in this hall. Before offering sacrifices at the Altar of the God of Agriculture, the emperor examined ceremonial farm tools here. After the revision of the imperial pedigree, which was revised once every ten years, the emperor read the pedigree out loud and held a grand ceremony at the hall. The words "Zhong He" come from the *Book of Rites*, meaning "When we handle matters properly and harmoniously without leaning to either side, all things on earth will flourish."

MADE POSSIBLE BY THE AMERICAN EXPRESS COMPANY

Decision Ranking

Risk is a characteristic of an activity relating to all possible events n_E which may follow as a result of the activity

The risk contribution R_{E_i} from the event E_i is defined through the product between
the event probability P_{E_i}

and

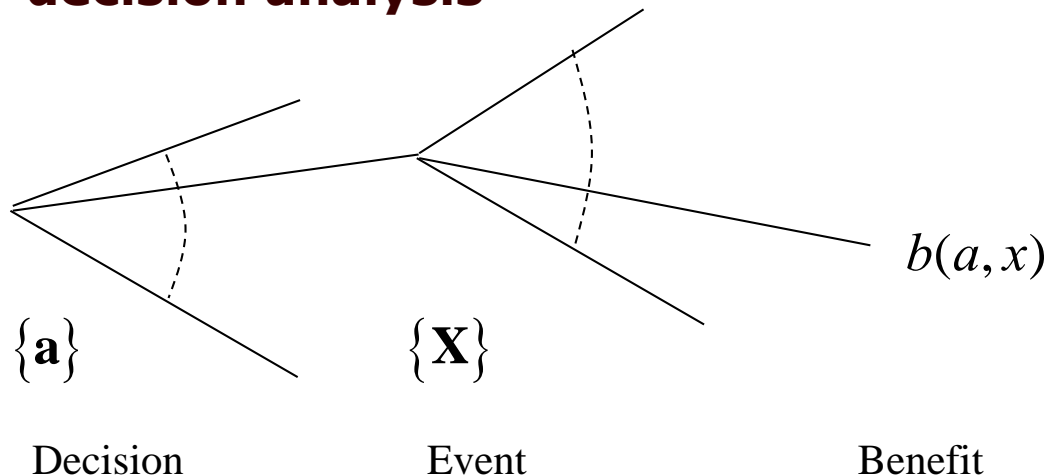
the consequences of the event C_{E_i}

The risk associated with a given activity R_A may then be written as

$$R_A = \sum_{i=1}^{n_E} R_{E_i} = \sum_{i=1}^{n_E} P_{E_i} \cdot C_{E_i}$$

Decision Optimization

Prior decision analysis



Information is
bought by choice of
prior density

Optimal decision maximizes the expected value of utility (benefit)
(von Neumann & Morgenstern)

$$B_0^* = \max_a E' [b(a, X)] = \max_a \int b(a, x) f'_X(x, a) dx$$

Decision Optimization

Posterior decision analysis

By sampling information \mathbf{z} using an experiment e we may update the probabilistic description of X

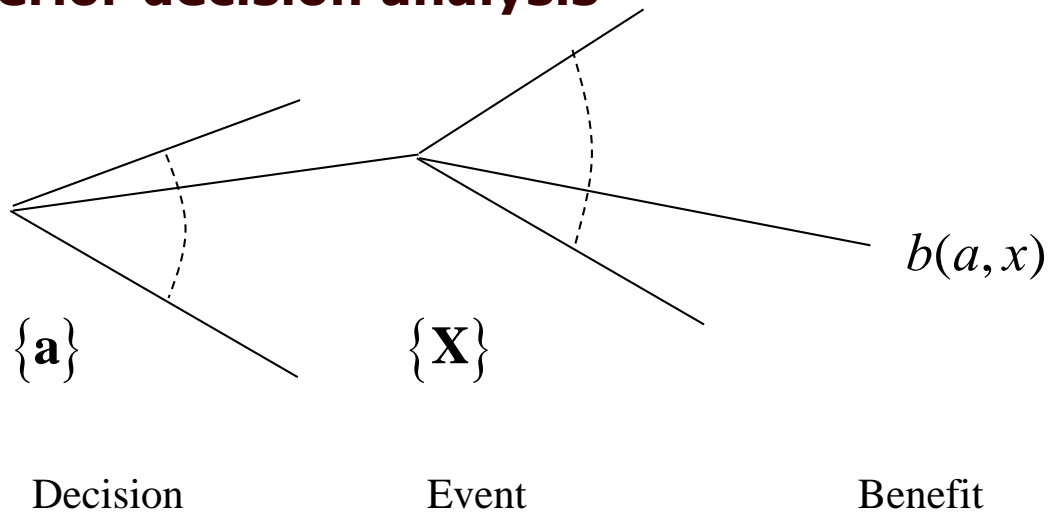
$$f_X''(x, a | \mathbf{z}) = \frac{L(x | \mathbf{z}) f_X'(x, a)}{\int L(x | \mathbf{z}) f_X'(x, a)}$$

Of course the likelihood of the sample \mathbf{z} depends on the experiment e why we write

$$L(x | \mathbf{z}) = L(x | \mathbf{z}, e)$$

Decision Optimization

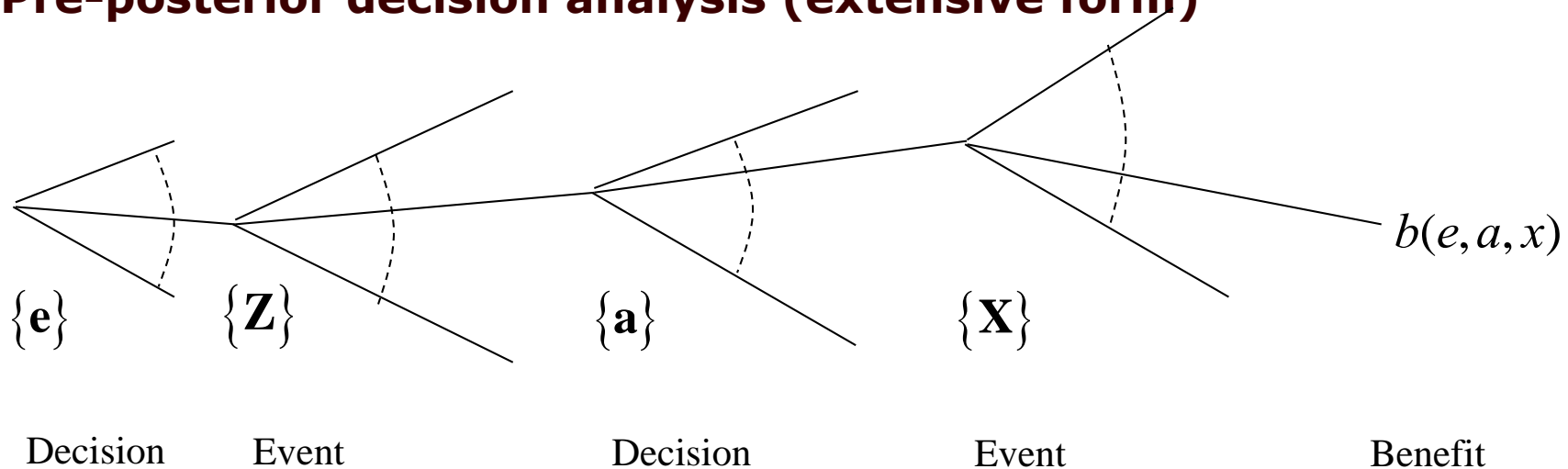
Posterior decision analysis



$$\max_a E''[b(a, X)] = \max_a \int b(a, x) f_X''(x, a | \hat{\mathbf{z}}) dx$$

Decision Optimization

Pre-posterior decision analysis (extensive form)

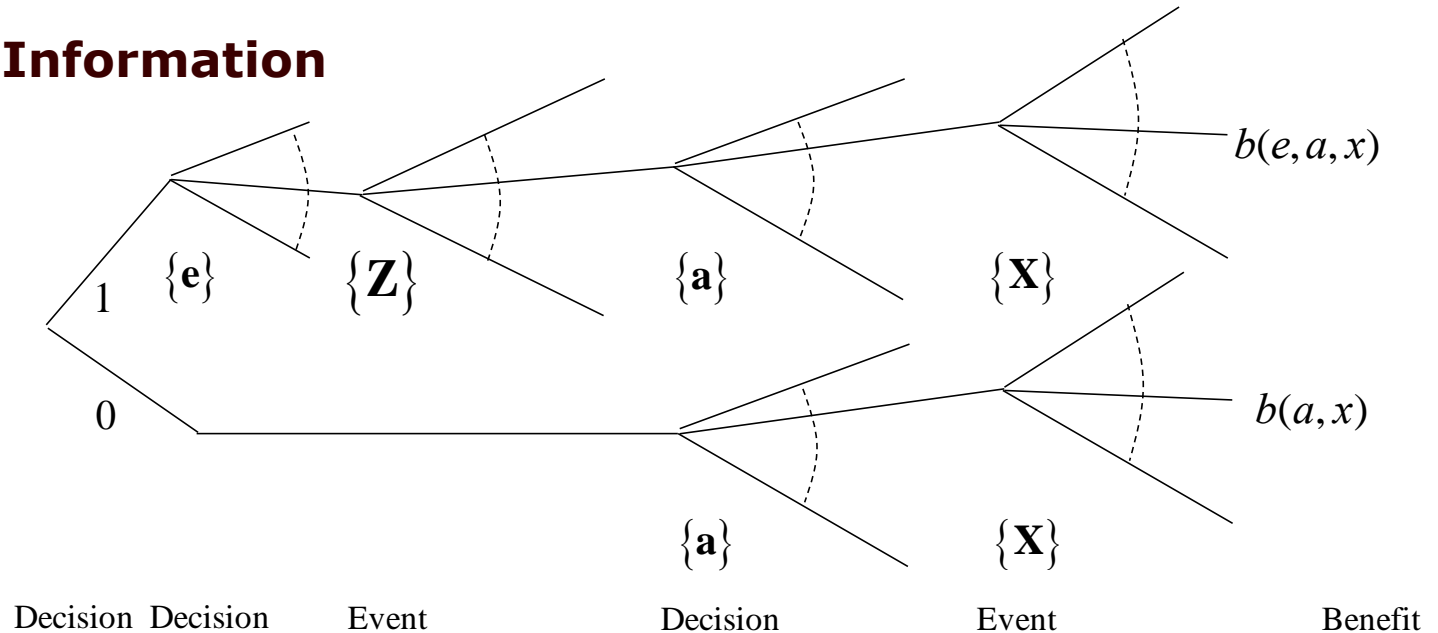


The optimal experiment e may be found from

$$B_1^* = \max_e E_Z \left[\max_a \int b(e, a, x) f_X''(x, a | Z) dx \right]$$

Decision Optimization

Value of Information



The value of information VoI is determined from:

$$VoI = \max_e E_Z \left[\max_a \int b(e, a, x) f_X''(x, a | \mathbf{Z}) dx \right] - \max_a \int b(a, x) f_X'(x, a) dx$$

Decision Optimization

Games and Risk

Rules (exogenous)

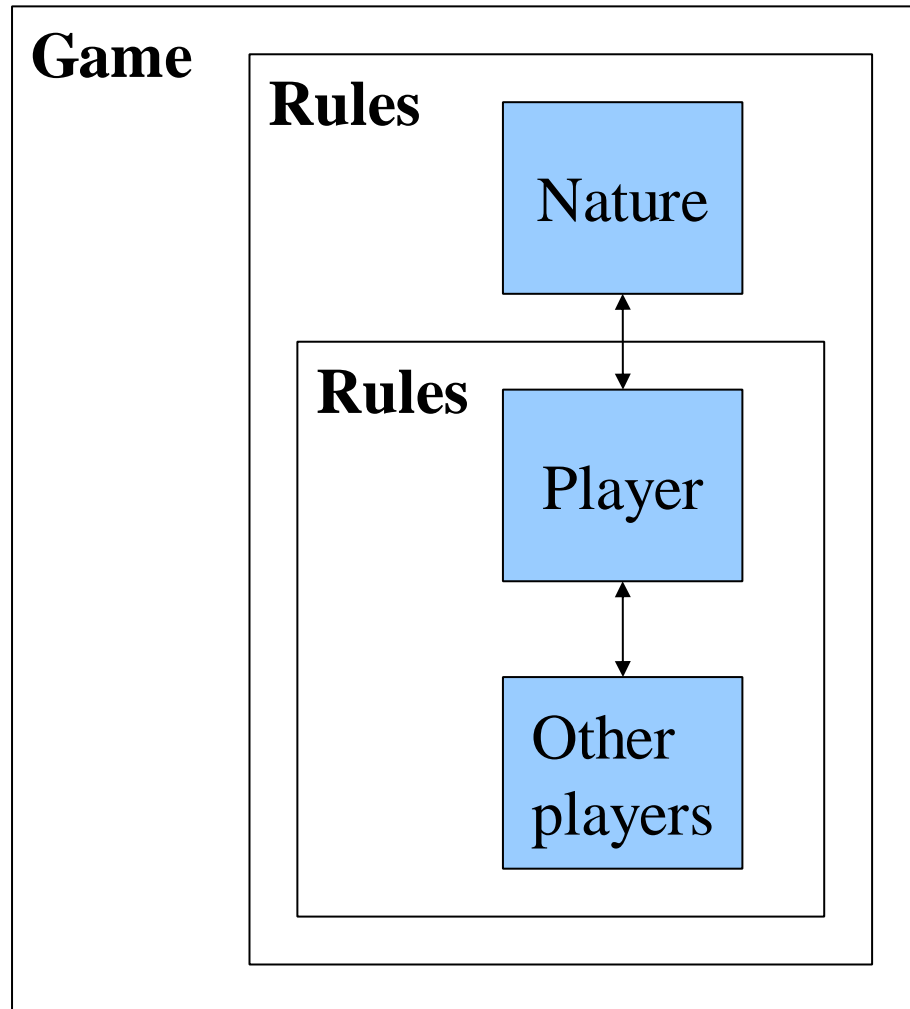
- Nature

Rules (endogenous)

- Knowledge
- Best practices
- Rules and standards
- Culture
- Ethics

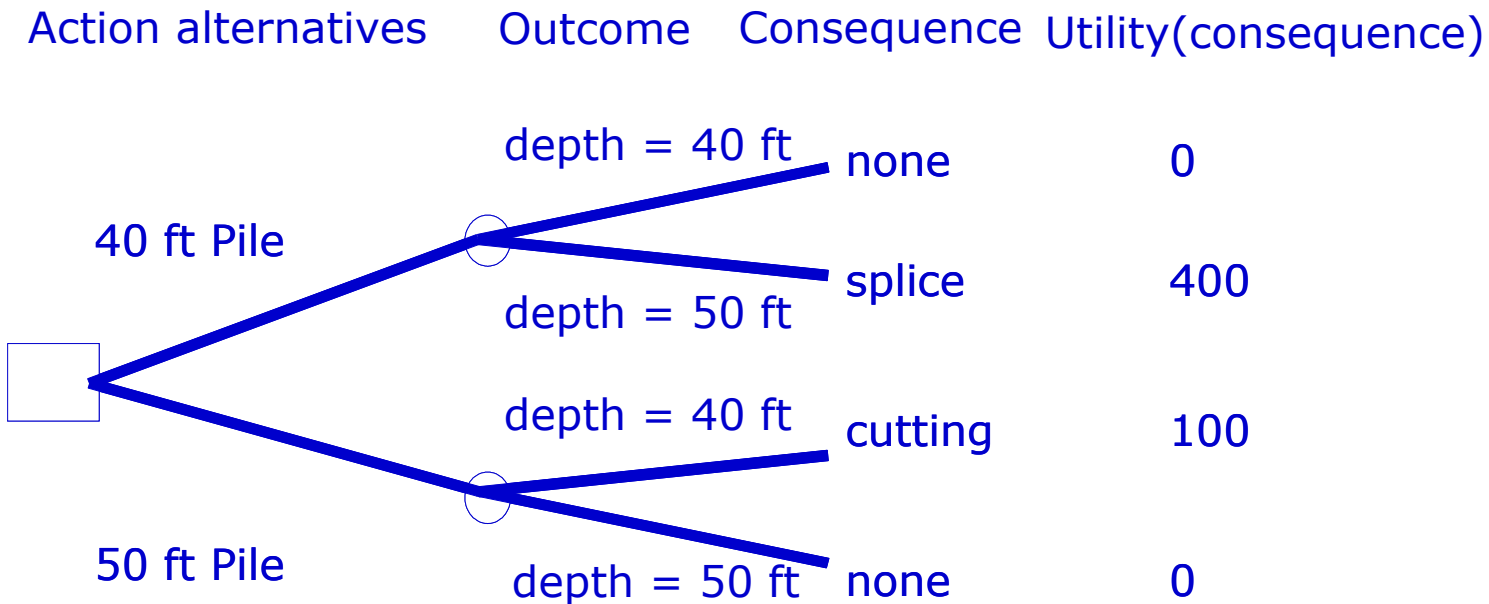
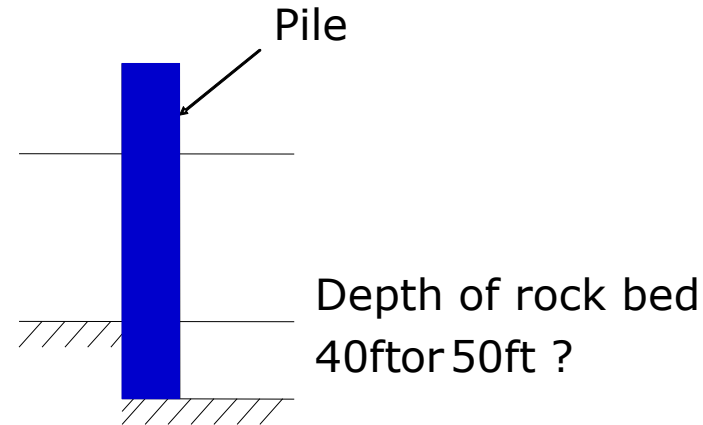
Drivers/Challenges

- Preferences
- Psychology
- Asymmetric information



Decision Analysis in Engineering

The decision tree



Decision Analysis in Engineering

The different types of decision analysis

- Prior
- Posterior
- Pre-posterior

Illustrated on an example :

Question : What pile length should be applied ?

Alternatives :

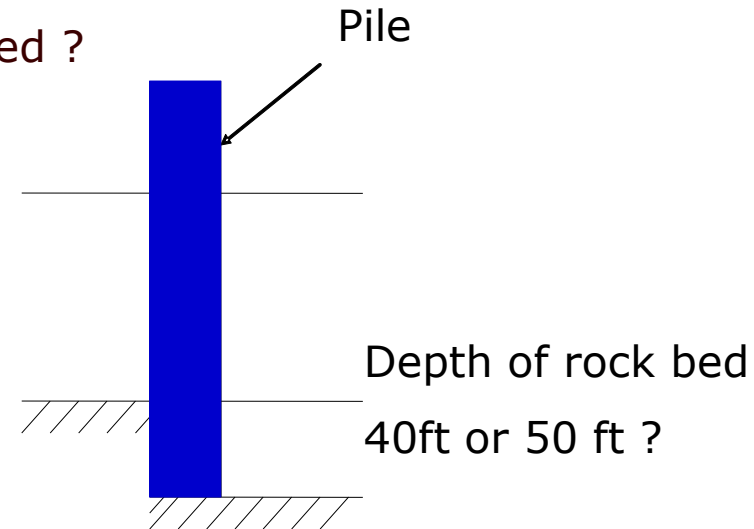
a_0 : Choose a 40 ft pile

a_1 : Choose a 50 ft pile

States of nature (depth to rock bed)

θ_0 : Rock bed at 40 ft

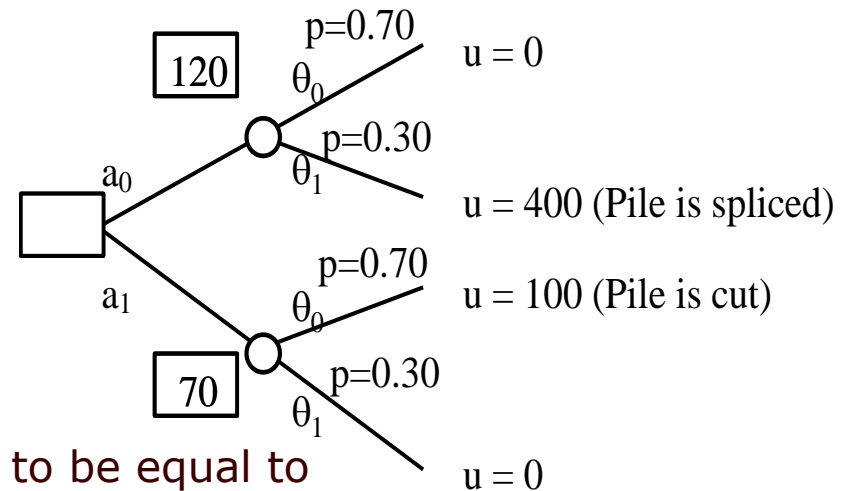
θ_1 : Rock bed at 50 ft



Decision Analysis in Engineering

Prior Analysis

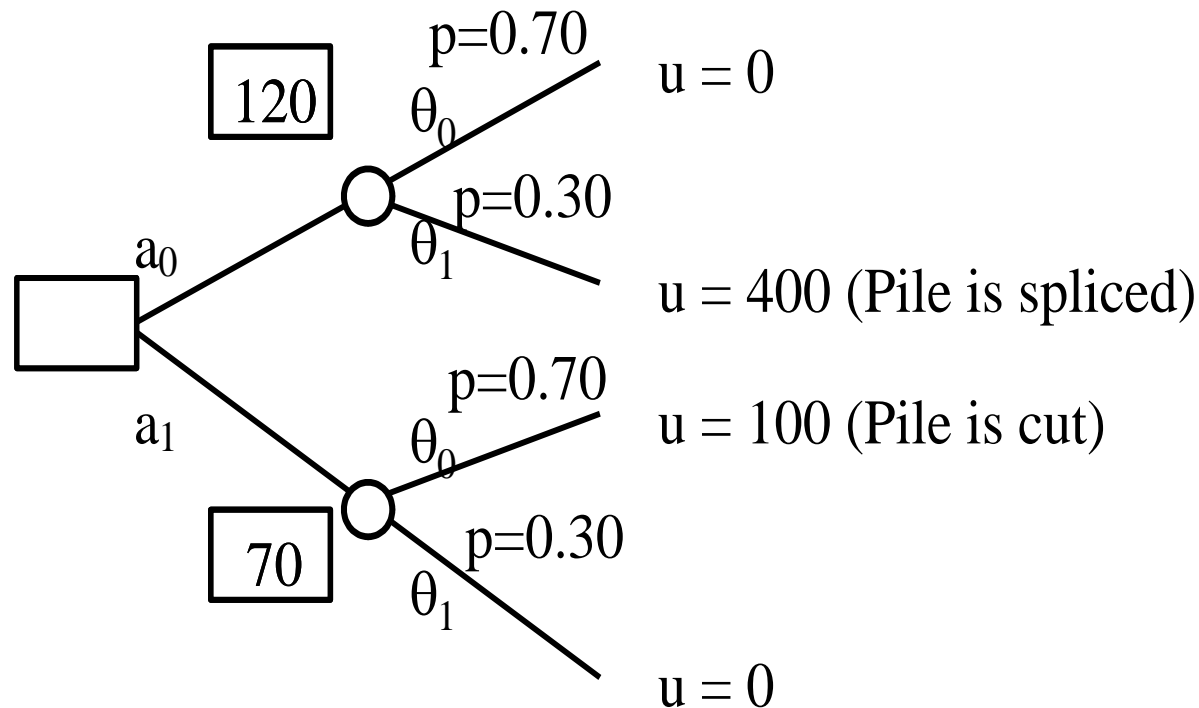
$$P'[\theta_0] = 0.70$$
$$P'[\theta_1] = 0.30$$



The expected utility is calculated to be equal to

$$E'[u] = \min\{u[a_0], u[a_1]\}$$
$$= \min\{P'[\theta_0] \times u[\theta_0|a_0] + P'[\theta_1] \times u[\theta_1|a_0],$$
$$P'[\theta_0] \times u[\theta_0|a_1] + P'[\theta_1] \times u[\theta_1|a_1]\}$$
$$= \min\{0.7 \times 0 + 0.3 \times 400, 0.7 \times 100 + 0.3 \times 0\}$$
$$= \min\{120, 70\} = 70 \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}$$

Decision Analysis in Engineering



\Rightarrow Choice of pile a_1 (50ft Pile)

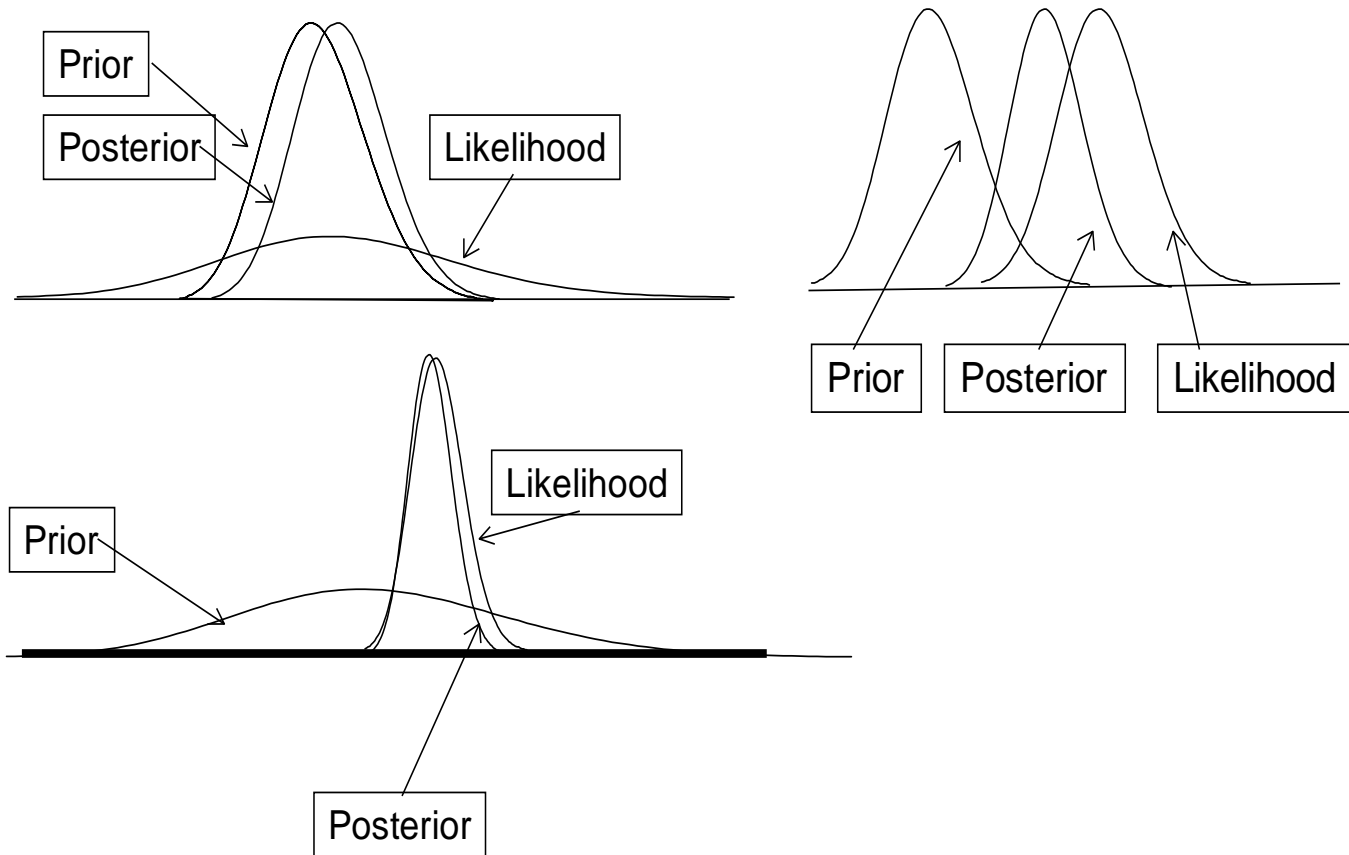
Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

Decision Analysis in Engineering

Posterior Analysis



Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

Ultrasonic tests to determine the depth to bed rock

True state \ Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P[z_k | \theta_j]$

Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P[z_k | \theta_i] P'[\theta_i]}{\sum_j P[z_k | \theta_j] P'[\theta_j]}$$

It is assumed that a test gives a 45 ft indication

$$P''[\theta_0] = P[\theta_0 | z_2] \propto P[z_2 | \theta_0] P[\theta_0] = 0.3 \times 0.7 = 0.21$$

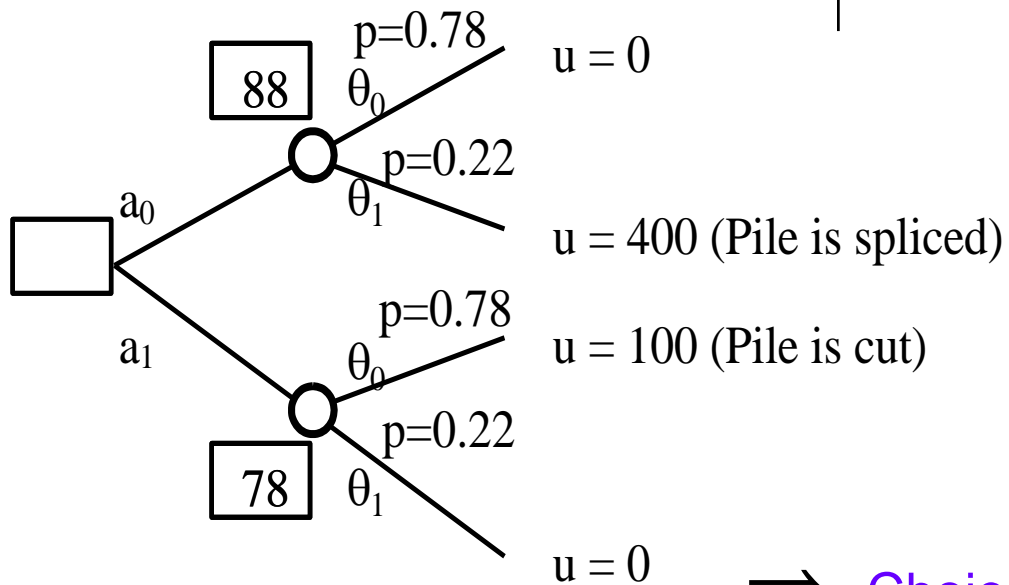
$$P''[\theta_1] = P[\theta_1 | z_2] \propto P[z_2 | \theta_1] P[\theta_1] = 0.2 \times 0.3 = 0.06$$

$$P''[\theta_0 | z_2] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''[\theta_1 | z_2] = \frac{0.06}{0.21 + 0.06} = 0.22$$

Decision Analysis in Engineering

Posterior Analysis

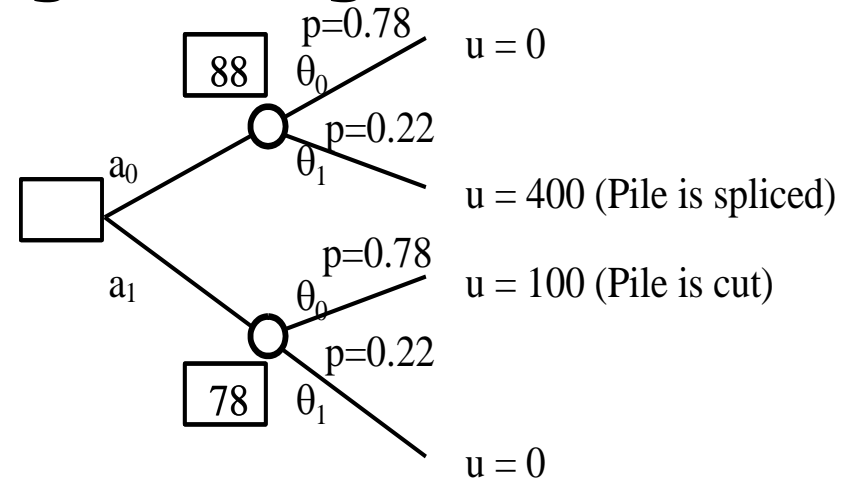


Test result indicates 45ft to rock bed

\Rightarrow Choice of alternative a_1 (50ft Pile)

Decision Analysis in Engineering

Posterior Analysis



$$E^*[u|z_2] = \min_j \{E^*[u(a_j)|z_2]\}$$

$$= \min\{P^*[\theta_0] \times 0 + P^*[\theta_1] \times 400, P^*[\theta_0] \times 100 + P^*[\theta_1] \times 0\}$$

$$= \min\{0.78 \times 0 + 0.22 \times 400, 0.78 \times 100 + 0.22 \times 0\}$$

$$= \min\{88, 78\} = 78$$

\Rightarrow Choice of alternative a_1 (50ft Pile)

Decision Analysis in Engineering

Pre-posterior Analysis

$$E[u] = \sum_{i=1}^n P'[z_i] \times E''[u|z_i] = \sum_{i=1}^n P'[z_i] \times \min_{j=1,m} \{E''[u(a_j)|z_i]\}$$

$$P'[z_i] = P[z_i|\theta_0] \times P'[\theta_0] + P[z_i|\theta_1] \times P'[\theta_1]$$

$$P'[z_0] = P[z_0|\theta_0] \times P'[\theta_0] + P[z_0|\theta_1] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

$$P'[z_1] = P[z_1|\theta_0] \times P'[\theta_0] + P[z_1|\theta_1] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P[z_2|\theta_0] \times P'[\theta_0] + P[z_2|\theta_1] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

Decision Analysis in Engineering

Pre-posterior Analysis

$$E^*[u|z_0] = \min_j \{E^*[u(a_j)|z_0]\}$$

$$\begin{array}{ccccccc} & & \mathbf{a_0} & & & \mathbf{a_1} & \\ & \underbrace{\hspace{10em}} & & \underbrace{\hspace{10em}} & & & \\ & \text{do nothing} & \text{splicing} & \text{cutting} & \text{do nothing} & & \\ = \min \{ & P^*[\theta_0|z_0] \times 0 + P^*[\theta_1|z_0] \times 400, & P^*[\theta_0|z_0] \times 100 + P^*[\theta_1|z_0] \times 0 \} \\ = \min \{ & 0.93 \times 0 + 0.07 \times 400, & 0.93 \times 100 + 0.07 \times 0 \} \\ = & 0.07 \times 400 + 0.93 \times 0 = 28 & & & & & \end{array}$$

Decision Analysis in Engineering

Pre-posterior Analysis

$$\begin{aligned} E^*[u|z_1] &= \min_j \{ E^*[u(a_j)|z_1] \} \\ &\quad \underbrace{\text{do nothing} \quad \text{splicing}}_{\mathbf{a}_0} \quad \underbrace{\text{cutting} \quad \text{do nothing}}_{\mathbf{a}_1} \\ &= \min \{ P^*[\theta_0|z_1] \times 0 + P^*[\theta_1|z_1] \times 400, P^*[\theta_0|z_1] \times 100 + P^*[\theta_1|z_1] \times 0 \} \\ &= \min \{ 0.25 \times 0 + 0.75 \times 400, 0.25 \times 100 + 0.75 \times 0 \} \\ &= 0.25 \times 100 + 0.75 \times 0 = 25 \end{aligned}$$

Decision Analysis in Engineering

Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis
– not including costs of experiments

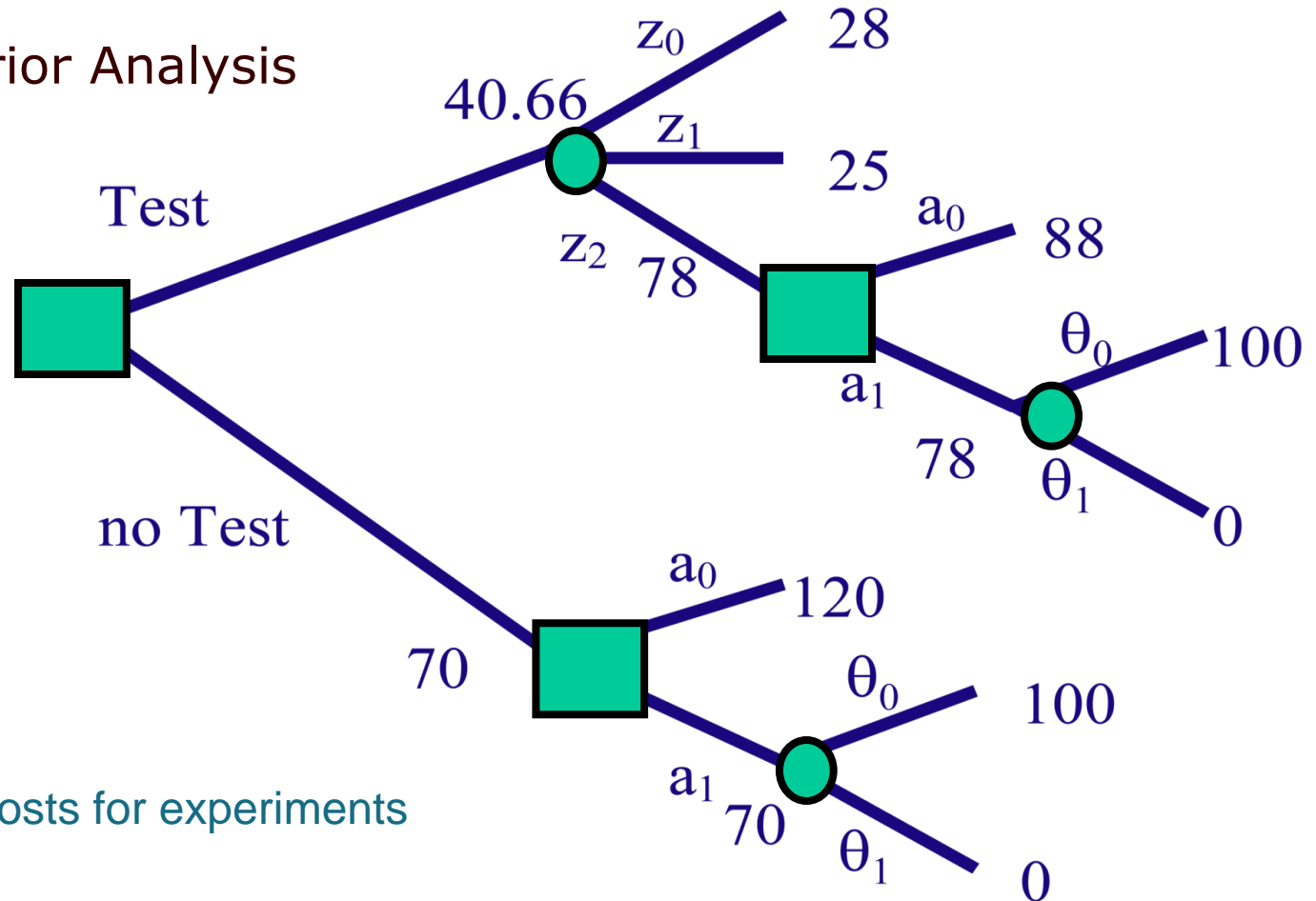
$$E[u] = \sum_{i=1}^n P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.00$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$

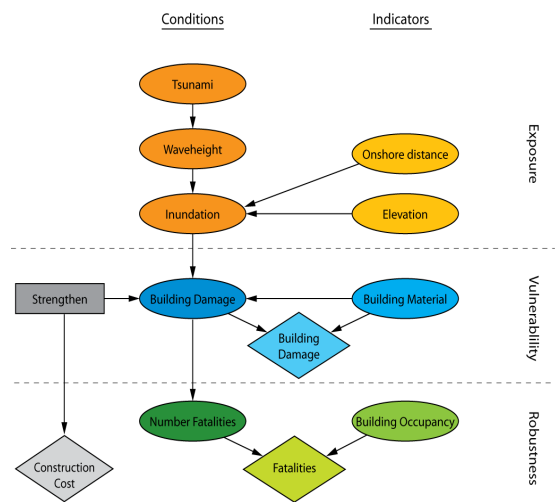
Decision Analysis in Engineering

Pre-posterior Analysis



Allowable costs for experiments

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$



Thanks for your attention 😊

