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Aalborg

Knowledge FOr Resilient soCiEty

Systems Engineering -
Decision Analysis in Engineering

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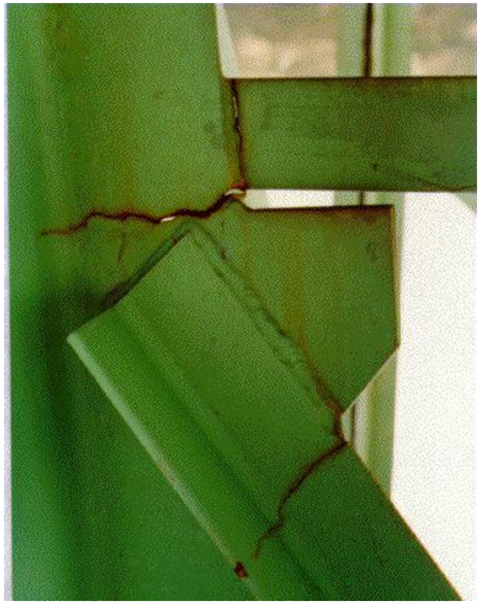
Contents of Presentation

- **Context of Engineering Decision Making**
- **Decisions and preferences**
- **Uncertainty**
- **Probability**
- **Decision Ranking**
- **Introduction to Decision Analysis**

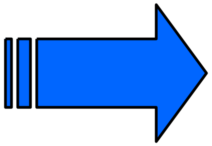
Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

What are we up against?



Context of Engineering Decision Making

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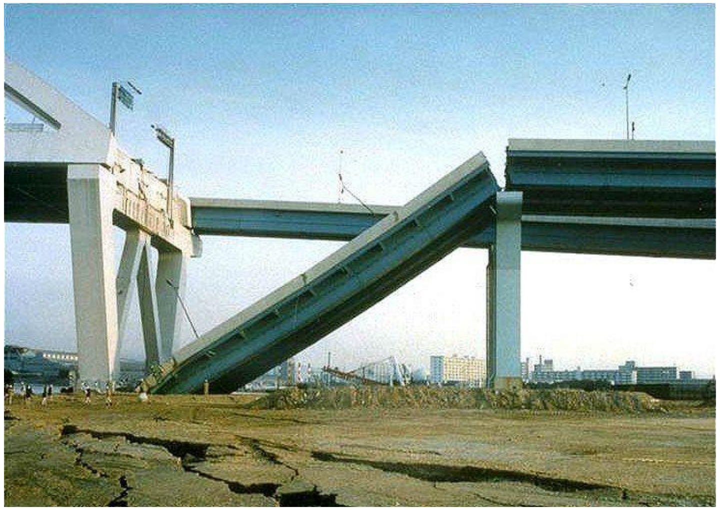

Context of Engineering Decision Making

What are we up against?

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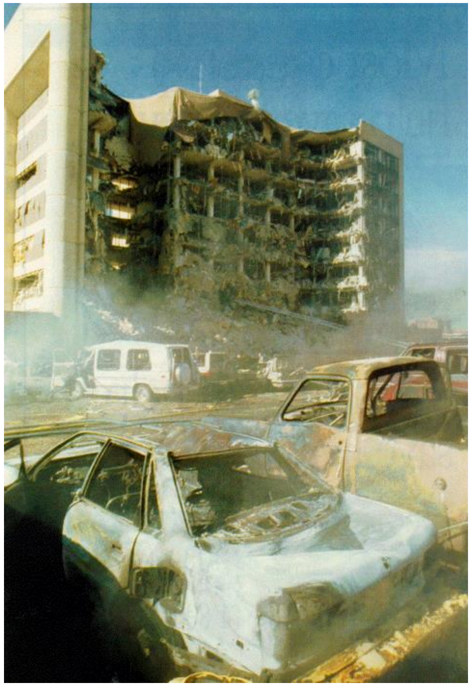


Context of Engineering Decision Making

What are we up against?

Deepwater Horizon
April 20, 2010

11 fatalities
17 injured
Oil spill > 5 million barrels
Health effects?
Eco. imp. > 10 billion $US
BP response – 14 billion $US
22000 lost jobs

Context of Engineering Decision Making

What are we up against?

Hurricane Katrina
August 23, 2005

> 1800 fatalities

Eco. imp. > 80 billion $US

Context of Engineering Decision Making

What are we up against?

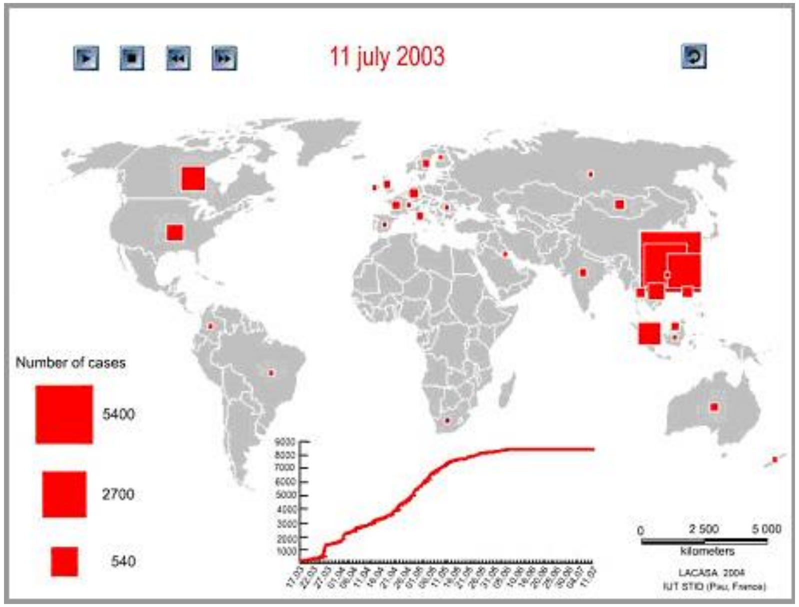
Fukushima Nuclear Event
March 11, 2011

No fatalities ..?

Eco. imp. > 75 billion $US

Context of Engineering Decision Making

What are we up against?



SARS, 2003
Fatalities: < 800
Eco. imp. 2% GDP – 200 billion $US

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Context of Engineering Decision Making

What are we up against?

Food borne diseases - USA
Affects 76 million per year
Hospitalizations: 325000 per year
Fatalities: 5000 pr year

Decisions and Preferences

Attributes of decision outcomes

Decisions aim to achieve an objective

The degree of achievement is measured by attributes

- natural attributes (measurable, e.g. costs and loss of lives)
- constructed attributes (a function of natural attributes e.g. GDP)
- proxy attributes (indicators which measure the perceived degree of fulfilment of an objective)

Decisions and Preferences

Preferences among attributes - utility

The attributes associated with a decision outcome may be translated into a degree of achievement of the objective by means of a utility function

different attributes are brought together on one or several scales

multi attribute decision making implies a weighing of different attributes

Decisions and Preferences

Constraints on decision making

In principle – any society may define what they consider to be acceptable decisions

Typically decisions are constrained – e.g. in terms of maximum acceptable risks to

- persons
- qualities of the environment

Uncertainty

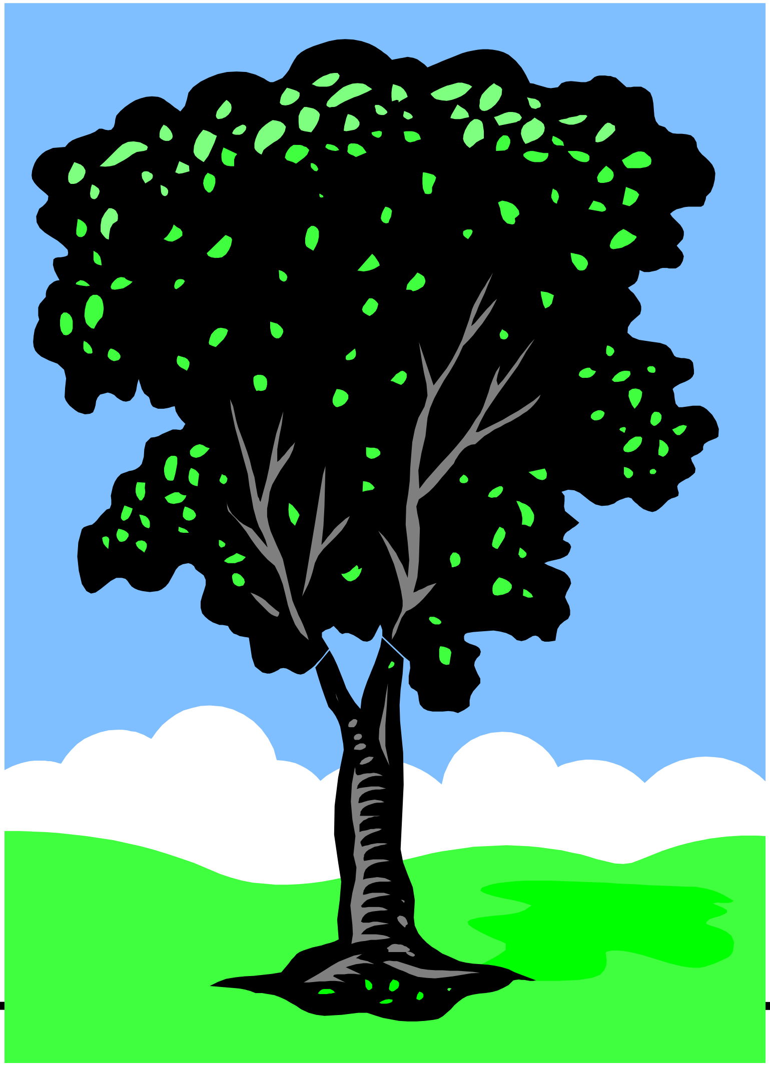
Different types of uncertainties influence decision making

Inherent natural variability – aleatory uncertainty
- result of throwing dices
- variations in material properties
- variations of wind loads
- variations in rain fall

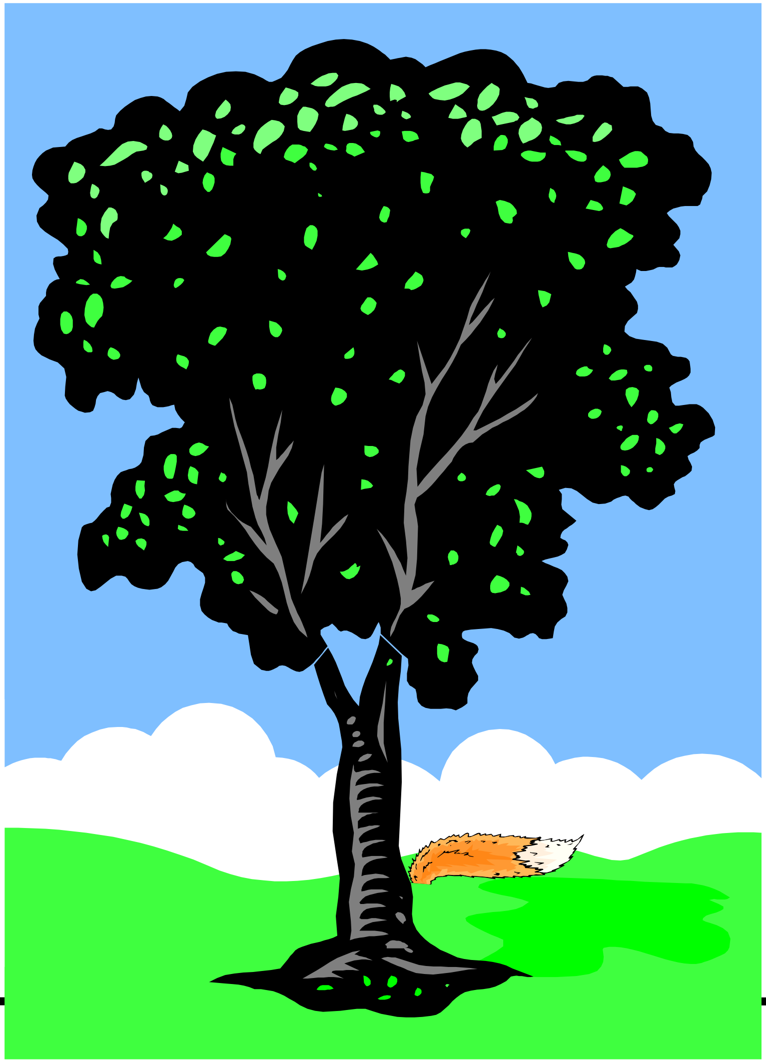
Model uncertainty – epistemic uncertainty
- lack of knowledge (future developments)
- inadequate/imprecise models (simplistic physical modelling)

Statistical uncertainties – epistemic uncertainty
- sparse information/small number of data

Probability



Probability



Probability

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Probability

Formulate hypothesis about the world

Utilize existing knowledge

Combine with data

Learn how to develop knowledge !

Probability

Conditional probabilities are of special interest as they provide the basis for utilizing new information in decision making.

The conditional probability of an event E_1 given that event E_2 has occured is written as:

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Not defined if $P(E_2) = 0$

The event E_1 is said to be probabilistically independent of the event E_2 if:

$$P(E_1 | E_2) = P(E_1)$$

Probability

From $P(E_1 | E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)}$

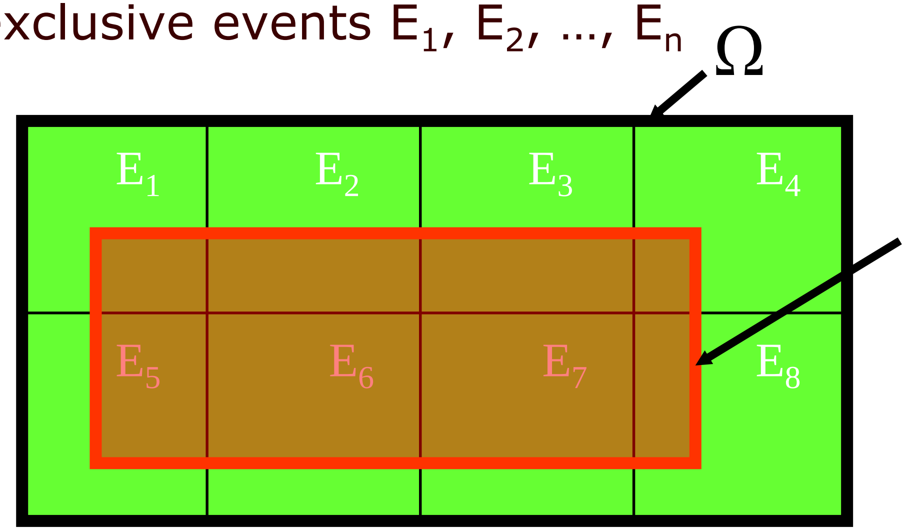
it follows that $P(E_1 \cap E_2) = P(E_2)P(E_1 | E_2)$

and when E_1 and E_2 are statistically independent it is

$$P(E_1 \cap E_2) = P(E_2)P(E_1)$$

Probability

Consider the sample space Ω divided up into n mutually exclusive events $E_1, E_2, \ldots, E_n$



$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_n)$$

$$P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \ldots + P(A|E_n)P(E_n) =$$

$$\sum_{i=1}^{n} P(A|E_i)P(E_i)$$

Probability

as there is $P(A \cap E_i) = P(A|E_i)P(E_i) = P(E_i|A)P(A)$

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

Posterior

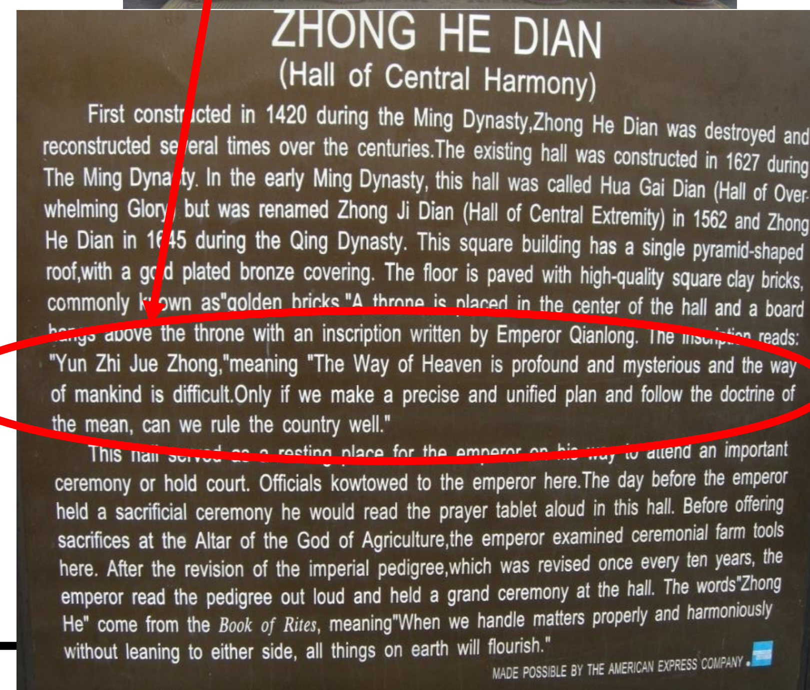
Bayes' Rule

Decision Ranking

Emperor Qianlong
Qing dynasty
Reign :1735 – 1796

Daniel Bernoulli 1738
Expected utility hypothesis

von Neumann and Morgenstern 1947
4 Axioms of utility theory:
Ranking based on expected value
of utility (VNM rational)



Decision Ranking

Risk is a characteristic of an activity relating to all possible events n_E which may follow as a result of the activity

The risk contribution R_{E_i} from the event E_i is defined through the product between

the event probability P_{E_i}

and

the consequences of the event C_{E_i}

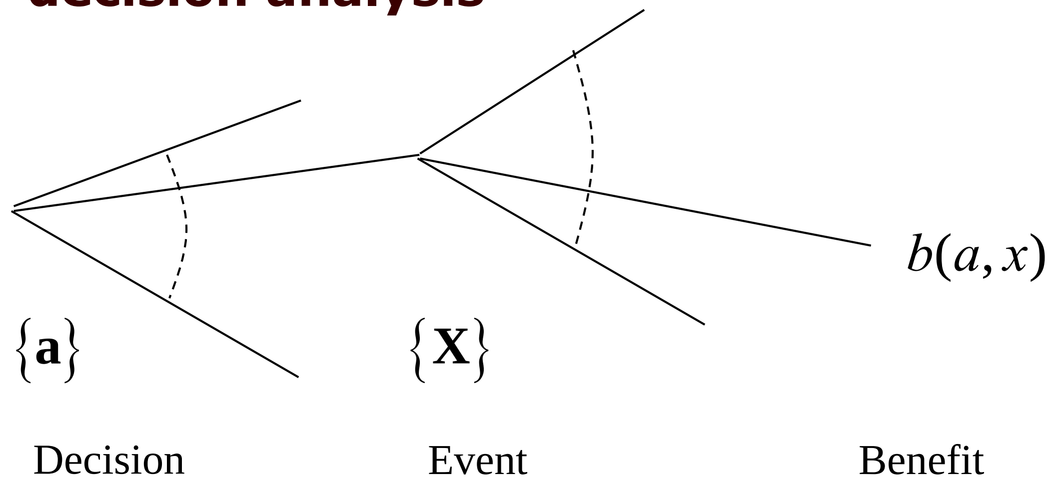
The risk associated with a given activity R_A may then be written as

$$R_A = \sum_{i=1}^{n_E} R_{E_i} = \sum_{i=1}^{n_E} P_{E_i} \cdot C_{E_i}$$

Decision Optimization

Prior decision analysis

Information is bought by choice of prior density



Optimal decision maximizes the expected value of utility (benefit)
(von Neumann & Morgenstern)

$$B_0^* = \max_a E'\left[b(a,X)\right] = \max_a \int b(a,x) f_X'(x,a)dx$$

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Decision Optimization

Posterior decision analysis

By sampling information $\mathbf{z}$ using an experiment e we may update the probabilistic description of X

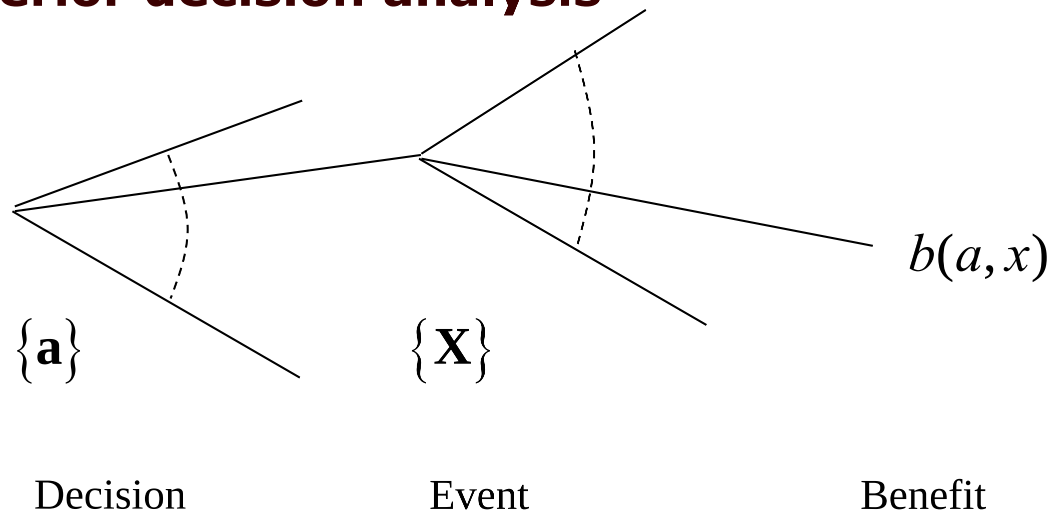
$$f''_X(x,a|\mathbf{z}) = \frac{L(x|\mathbf{z})f'_X(x,a)}{\int L(x|\mathbf{z})f'_X(x,a)}$$

Of course the likelihood of the sample $\mathbf{z}$ depends on the experiment e why we write

$$L(x|\mathbf{z}) = L(x|\mathbf{z},e)$$

Decision Optimization

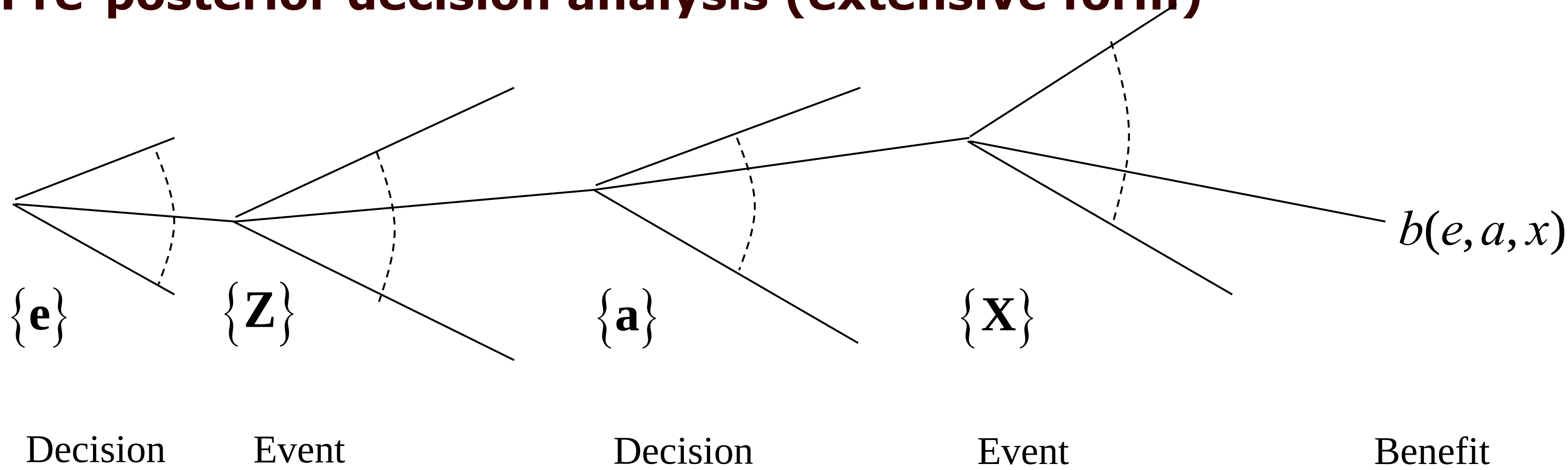
Posterior decision analysis



$$\max_a E''\left[b(a,X)\right] = \max_a \int b(a,x) f''_X(x,a\,|\,\hat{\mathbf{z}})dx$$

Decision Optimization

Pre-posterior decision analysis (extensive form)

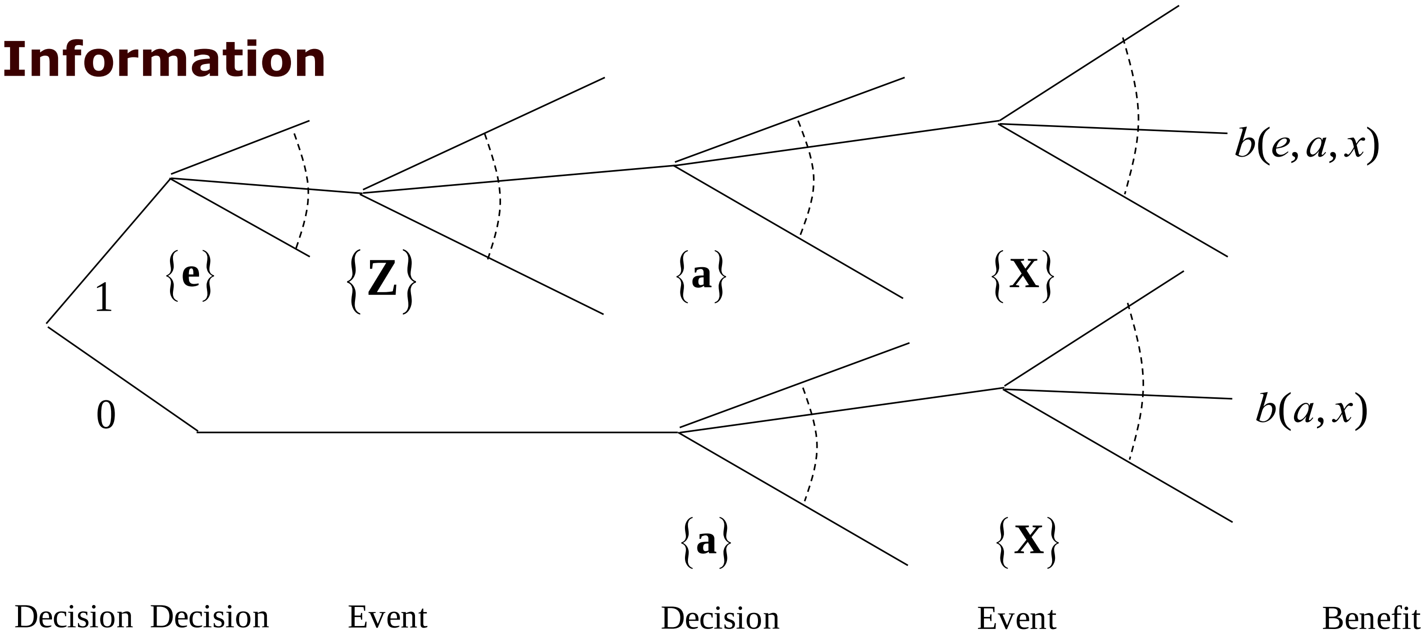


The optimal experiment e may be found from

$$B_1^* = \max_e E_{\mathbf{Z}}\left\lfloor \max_a \int b(e,a,x) f_X''(x,a\,|\,\mathbf{Z})dx \right\rfloor$$

Decision Optimization

Value of Information



The value of information VoI is determined from:

$$VoI = \max_{e} E_{\mathbf{Z}}\left[\max_{a} \int b(e,a,x) f_X''(x,a|\mathbf{Z})dx\right] - \max_{a} \int b(a,x) f_X'(x,a)dx$$

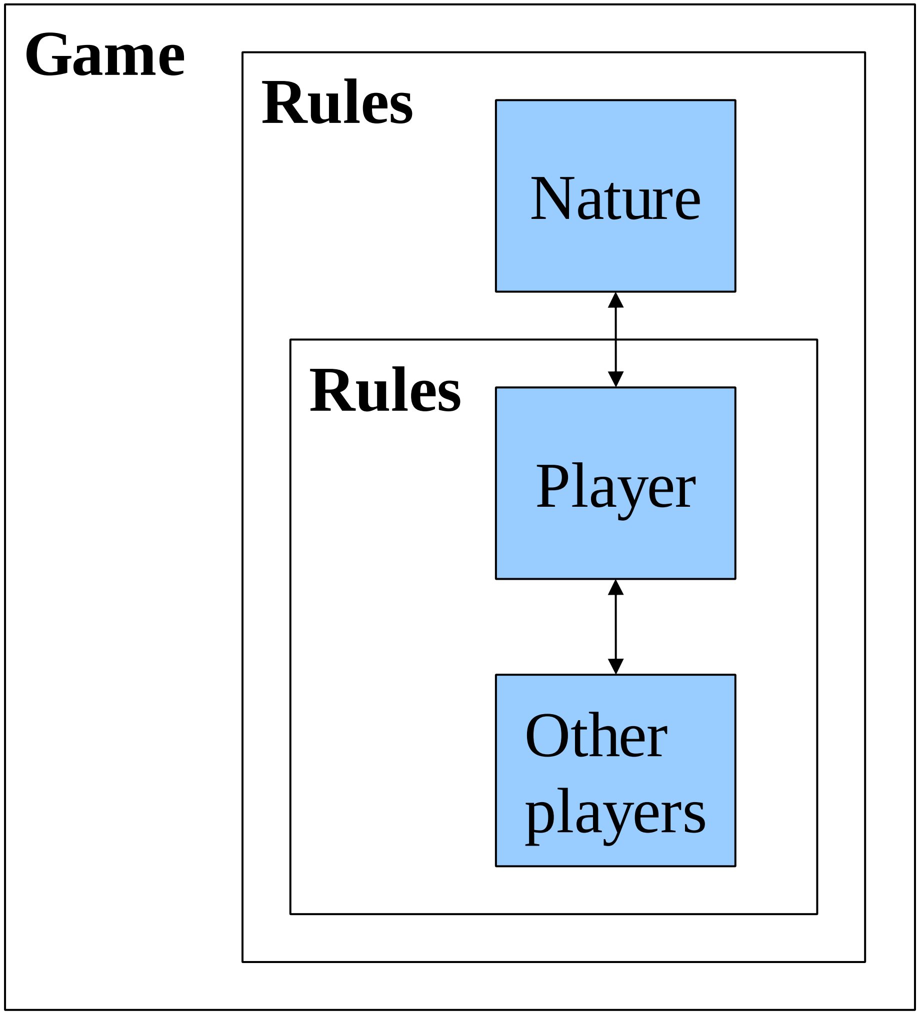
Decision Optimization

Games and Risk

Rules (exogenous)
- Nature

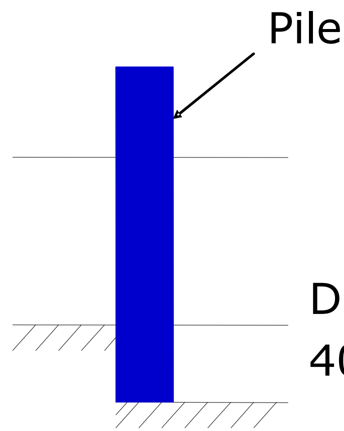
Rules (endogenous)
- Knowledge
- Best practices
- Rules and standards
- Culture
- Ethics

Drivers/Challenges
- Preferences
- Psychology
- Asymmetric information

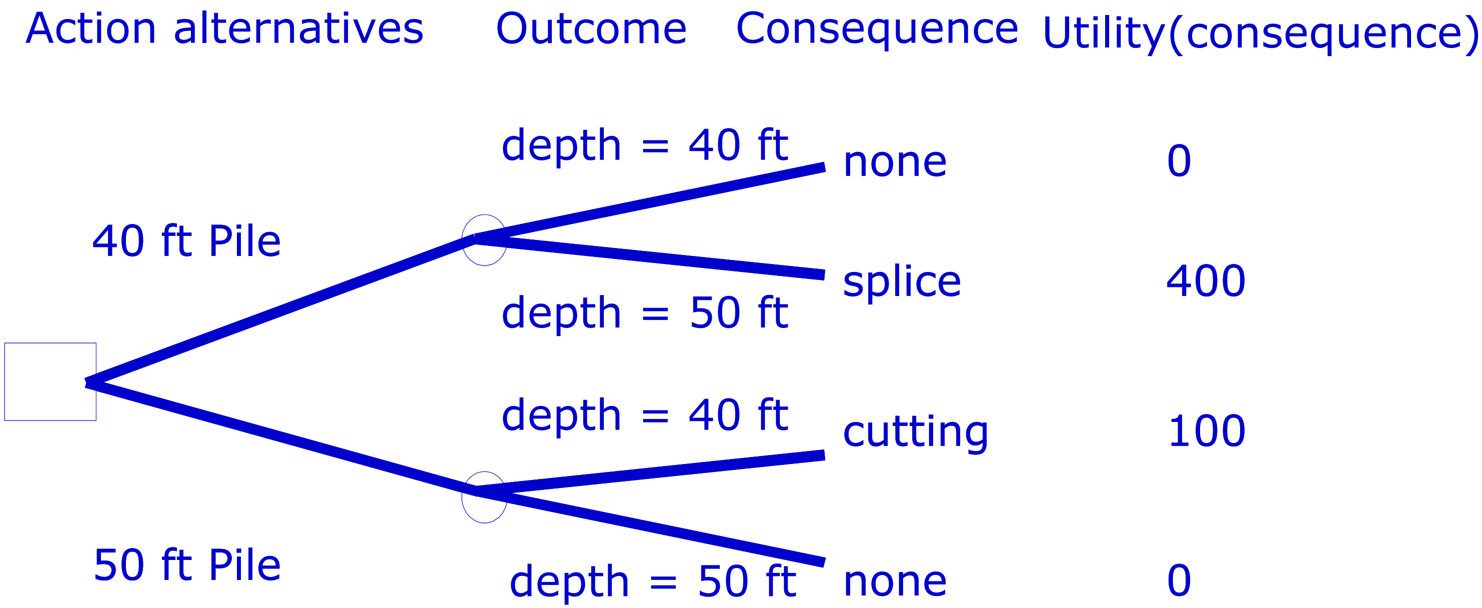


Decision Analysis in Engineering

The decision tree



Depth of rock bed
40ft or 50ft ?



Decision Analysis in Engineering

The different types of decision analysis

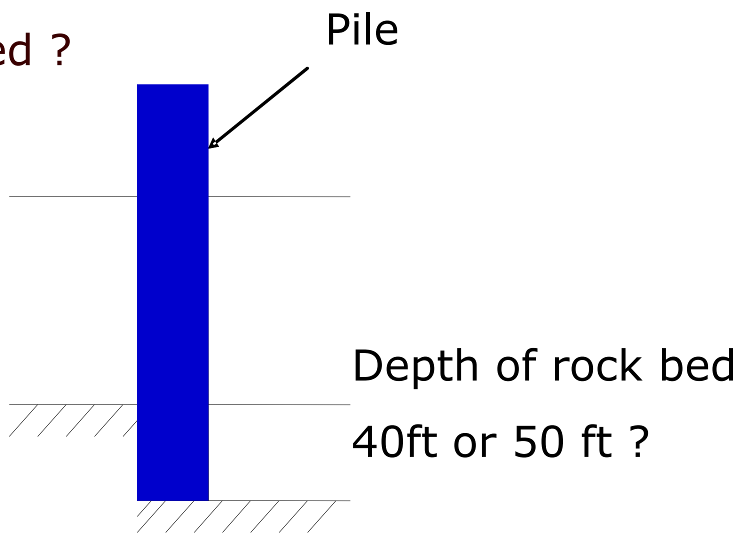
- Prior
- Posterior
- Pre-posterior

Illustrated on an example :

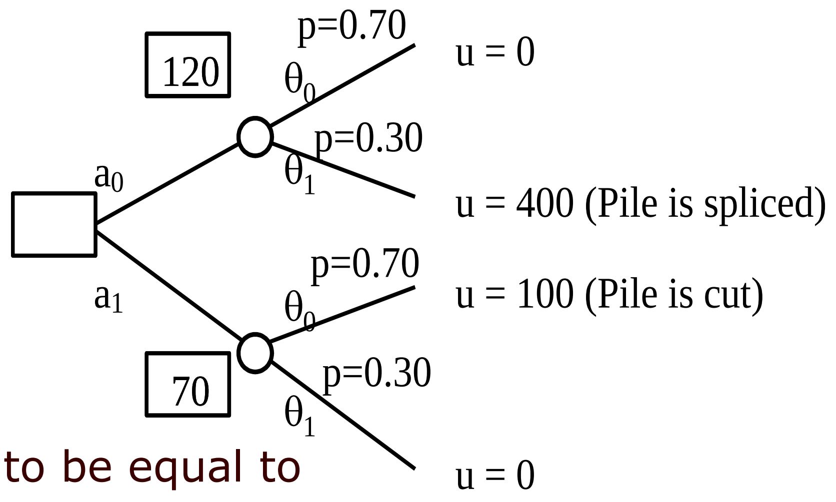
Question : What pile length should be applied ?

Alternatives :
a_0 : Choose a 40 ft pile
a_1 : Choose a 50 ft pile

States of nature (depth to rock bed)
θ_0 : Rock bed at 40 ft
θ_1 : Rock bed at 50 ft



Decision Analysis in Engineering



Prior Analysis

$P'[\theta_0] = 0.70$
$P'[\theta_1] = 0.30$

The expected utility is calculated to be equal to

$$E'[u] = \min\{u[a_0]\ , u[a_1]\}$$

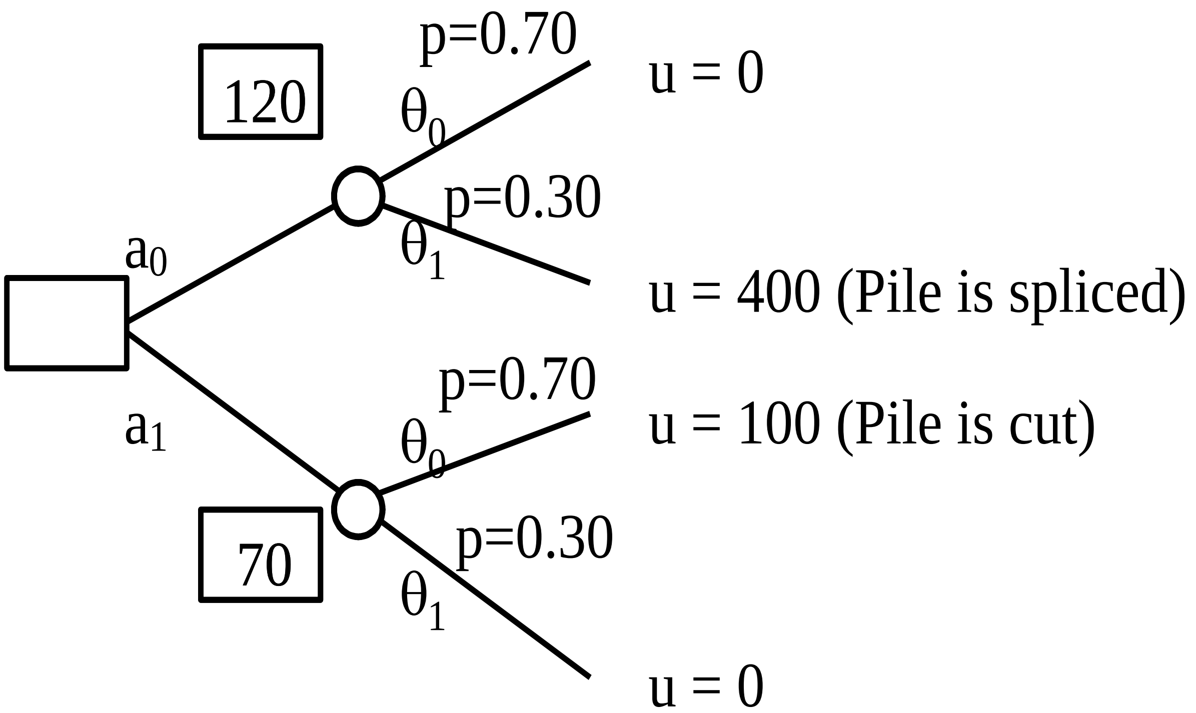
$$= \min\{P'[\theta_0]\times u[\theta_0|a_0] + P'[\theta_1]\times u[\theta_1|a_0]\ ,$$

$$P'[\theta_0]\times u[\theta_0|a_1] + P'[\theta_1]\times u[\theta_1|a_1]\}$$

$$= \min\{0.7\times 0 + 0.3\times 400,\ 0.7\times 100 + 0.3\times 0\}$$

$$= \min\{120, 70\} = 70 \quad \Rightarrow \text{Decision for } a_1 \text{ (50ft Pile)}$$

Decision Analysis in Engineering



$\Longrightarrow$ Choice of pile a_1 (50ft Pile)

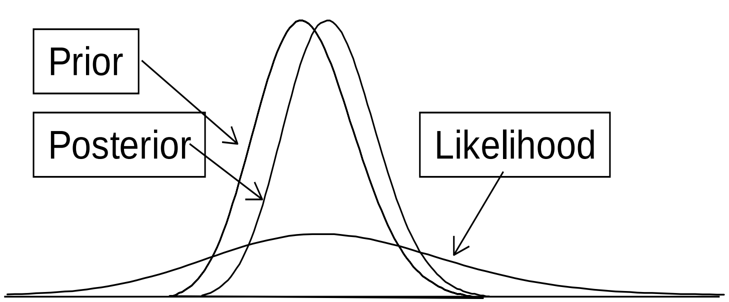
Decision Analysis in Engineering

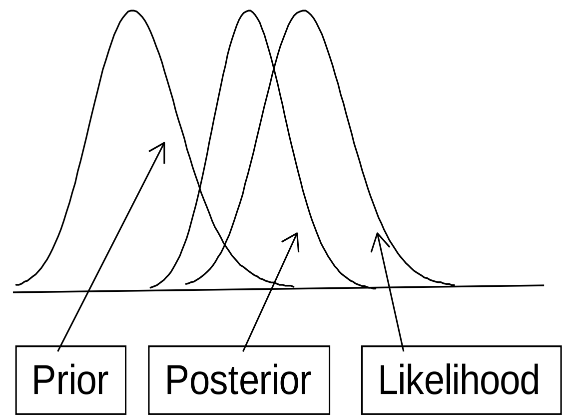
Posterior Analysis

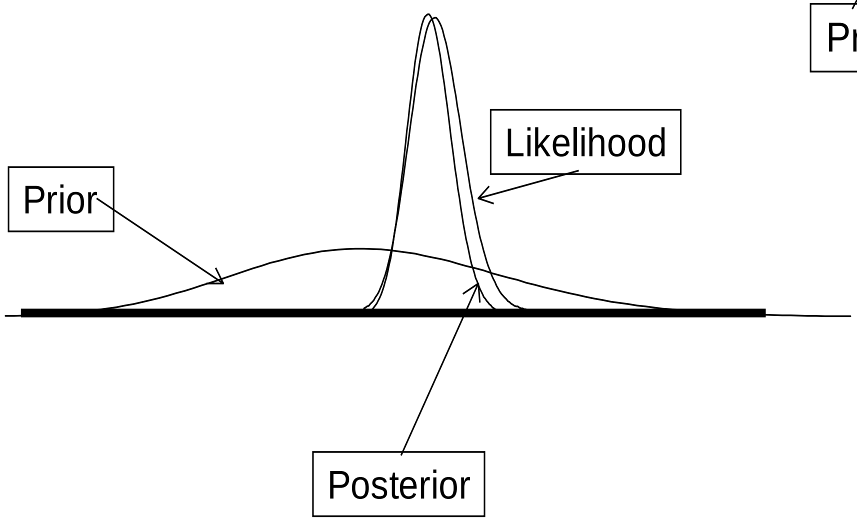
$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

Decision Analysis in Engineering

Posterior Analysis







Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'[\theta_i]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

Ultrasonic tests to determine the depth to bed rock

True state / Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P\left[z_k \middle| \theta_j\right]$

Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

It is assumed that a test gives a 45 ft indication

$$P''\left[\theta_0\right] = P\left[\theta_0 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_0\right] P\left[\theta_0\right] = 0.3\ x\ 0.7 = 0.21$$

$$P''\left[\theta_1\right] = P\left[\theta_1 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_1\right] P\left[\theta_1\right] = 0.2\ x\ 0.3 = 0.06$$

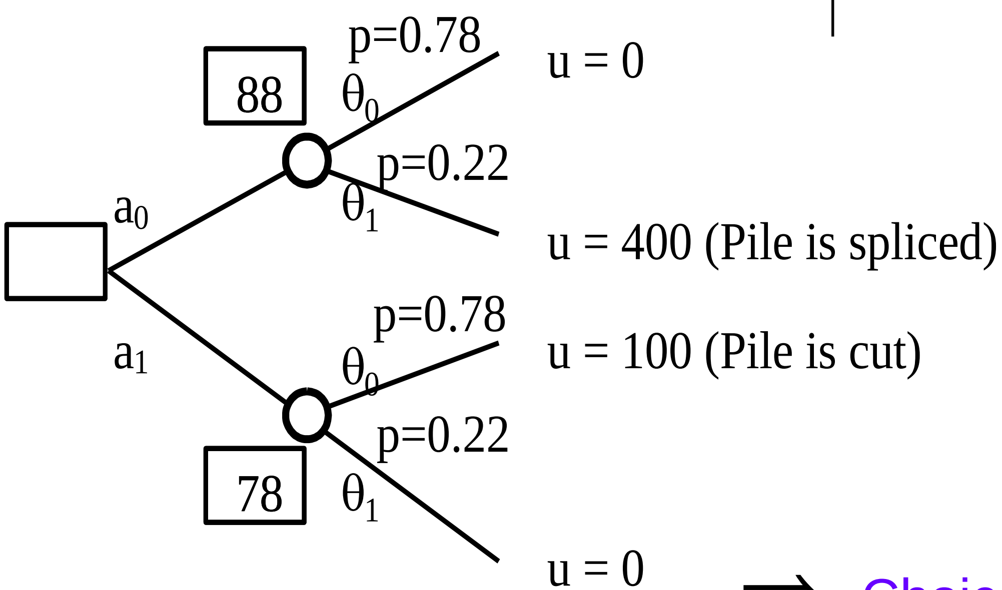
$$P''\left[\theta_0 \middle| z_2\right] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''\left[\theta_1 \middle| z_2\right] = \frac{0.06}{0.21 + 0.06} = 0.22$$

Decision Analysis in Engineering

Posterior Analysis

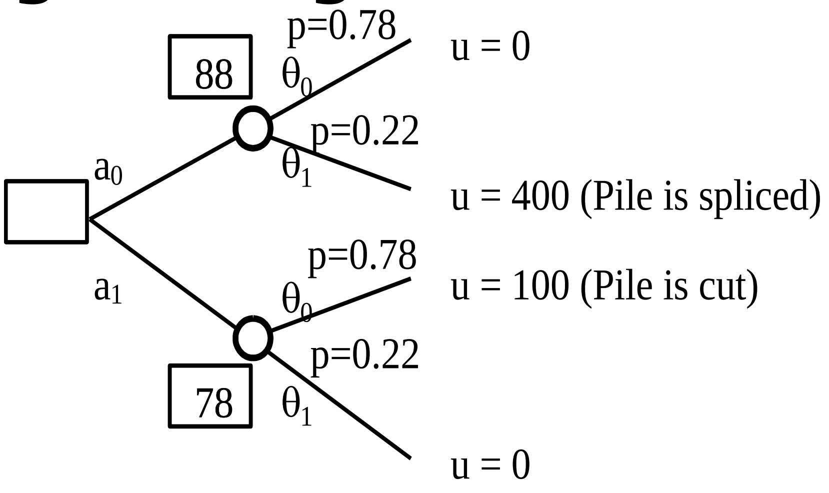
Test result indicates 45ft to rock bed



$\Rightarrow$ Choice of alternative a_1 (50ft Pile)

Decision Analysis in Engineering

Posterior Analysis



$$E''\left[u \middle| z_2\right] = \min_j \{E''\left[u(a_j) \middle| z_2\right]\}$$

$$= \min\{P''\left[\theta_0\right] \times 0 + P''\left[\theta_1\right] \times 400,\ P''\left[\theta_0\right] \times 100 + P''\left[\theta_1\right] \times 0\}$$

$$= \min\{0.78 \times 0 + 0.22 \times 400,\ 0.78 \times 100 + 0.22 \times 0\}$$

$$= \min\{88\ ,\ 78\} = 78$$

$\Rightarrow$ Choice of alternative a_1 (50ft Pile)

Decision Analysis in Engineering

Pre-posterior Analysis

$$E\left[u\right]=\sum_{i=1}^{n}P'\left[z_i\right]\times E''\left[u\,|\,z_i\right]=\sum_{i=1}^{n}P'\left[z_i\right]\times\min_{j=1,m}\{E''\left[u(a_j)\,|\,z_i\right]\}$$

$$P'\left[z_i\right]=P\left[z_i\,|\,\theta_0\right]\times P'\left[\theta_0\right]+P\left[z_i\,|\,\theta_1\right]\times P'\left[\theta_1\right]$$

$$P'\left[z_0\right]=P\left[z_0\,|\,\theta_0\right]\times P'\left[\theta_0\right]+P\left[z_0\,|\,\theta_1\right]\times P'\left[\theta_1\right]=0.6\times0.7+0.1\times0.3=0.45$$

$$P'\left[z_1\right]=P\left[z_1\,|\,\theta_0\right]\times P'\left[\theta_0\right]+P\left[z_1\,|\,\theta_1\right]\times P'\left[\theta_1\right]=0.1\times0.7+0.7\times0.3=0.28$$

$$P'\left[z_2\right]=P\left[z_2\,|\,\theta_0\right]\times P'\left[\theta_0\right]+P\left[z_2\,|\,\theta_1\right]\times P'\left[\theta_1\right]=0.3\times0.7+0.2\times0.3=0.27$$

Decision Analysis in Engineering

Pre-posterior Analysis

$$E''\left[u \middle| z_0\right] = \min_j\{E''\left[u(a_j) \middle| z_0\right]\}$$

$$\overbrace{\underset{\text{do nothing}}{} \quad \underset{\text{splicing}}{}}^{\mathbf{a_0}} \quad \overbrace{\underset{\text{cutting}}{} \quad \underset{\text{do nothing}}{}}^{\mathbf{a_1}}$$

$$= \min\{P''\left[\theta_0 \middle| z_0\right]\times 0 + P''\left[\theta_1 \middle| z_0\right]\times 400,\ P''\left[\theta_0 \middle| z_0\right]\times 100 + P''\left[\theta_1 \middle| z_0\right]\times 0\}$$

$$= \min\{0.93\times 0 + 0.07\times 400,\ 0.93\times 100 + 0.07\times 0\}$$

$$= 0.07\times 400 + 0.93\times 0 = 28$$

Decision Analysis in Engineering

Pre-posterior Analysis

$$E''\left[u\middle|z_1\right] = \min_j\{E''\left[u(a_j)\middle|z_1\right]\}$$

a_0: do nothing, splicing

a_1: cutting, do nothing

$$= \min\{P''\left[\theta_0\middle|z_1\right]\times 0 + P''\left[\theta_1\middle|z_1\right]\times 400,\ P''\left[\theta_0\middle|z_1\right]\times 100 + P''\left[\theta_1\middle|z_1\right]\times 0\}$$

$$= \min\{0.25\times 0 + 0.75\times 400,\ 0.25\times 100 + 0.75\times 0\}$$

$$= 0.25\times 100 + 0.75\times 0 = 25$$

Decision Analysis in Engineering

Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis
– not including costs of experiments

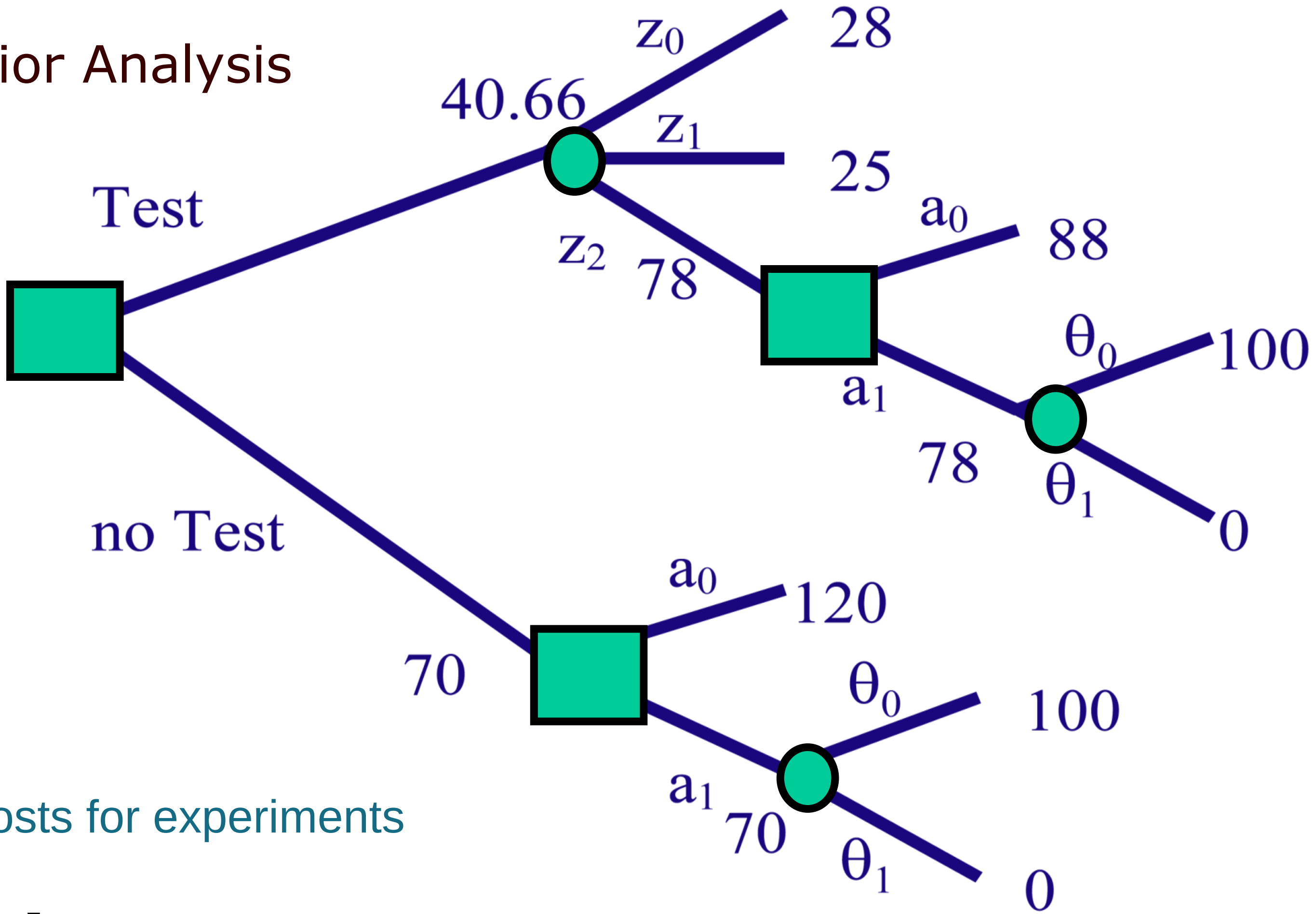
$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.00$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$

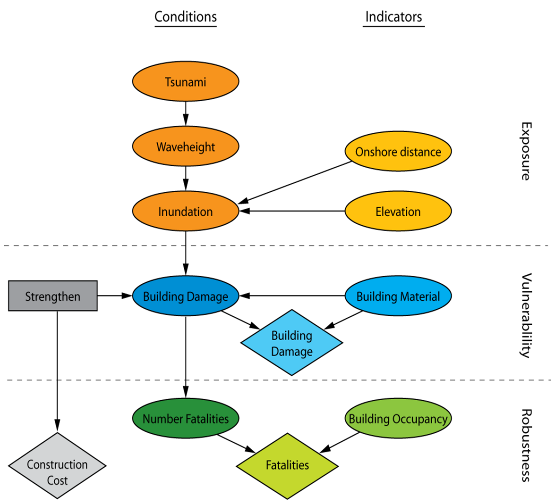
Decision Analysis in Engineering

Pre-posterior Analysis



Allowable costs for experiments

$$E'[u]-E[u]=70.00-40.00=30.00$$



Thanks for your attention ☺