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Place: Novi Sad, Serbia

Knowledge FOr Resilient soCiEty

Basics of Decision Analysis in Engineering

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Aalborg University

Contents of Presentation

- **Context of Engineering Decision Making**
- **Decisions and preferences**
- **Uncertainty**
- **Probability**
- **Decision Ranking**
- **Decision analysis in engineering**

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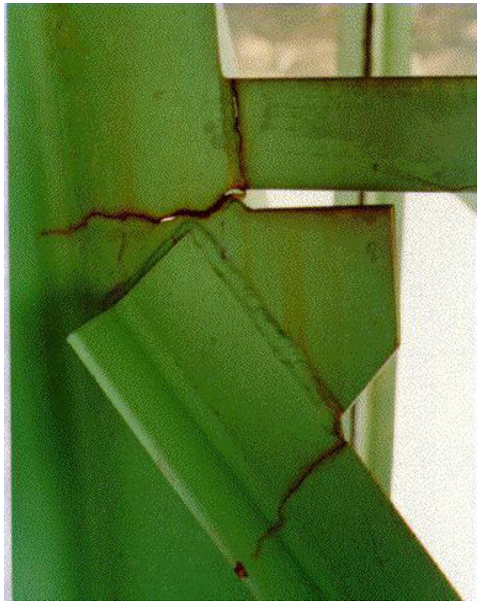
Context of Engineering Decision Making

What are we up against?

Corrosion



Fatigue

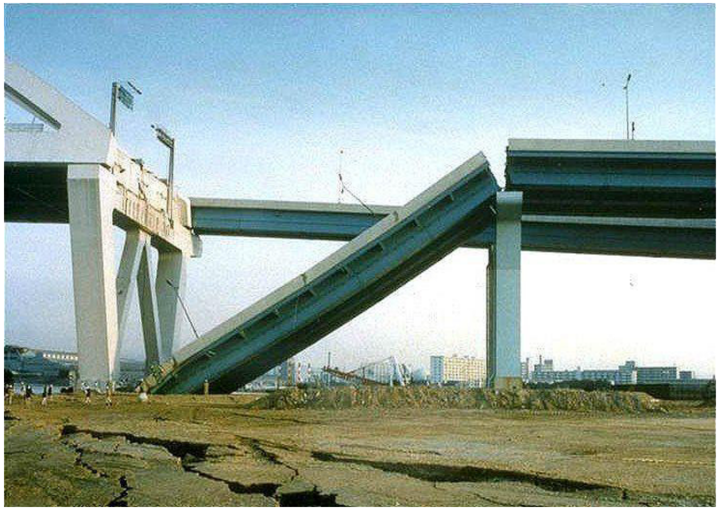
Context of Engineering Decision Making

What are we up against?

Tornados and strong winds

Context of Engineering Decision Making

What are we up against?



Earthquakes

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Context of Engineering Decision Making

What are we up against?

Earth slide

Rock fall

Context of Engineering Decision Making

What are we up against?

Fires

Explosions

Context of Engineering Decision Making

What are we up against?

Over load

Design error

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Context of Engineering Decision Making

What are we up against?

Bombs

Airplane impacts

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Context of Engineering Decision Making

What are we up against?

Deepwater Horizon
April 20, 2010

11 fatalities
17 injured
Oil spill > 5 million barrels
Health effects?
Eco. imp. > 10 billion $US
BP response – 14 billion $US
22000 lost jobs

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Context of Engineering Decision Making

What are we up against?

Hurricane Katrina
August 23, 2005

> 1800 fatalities

Eco. imp. > 80 billion $US

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Context of Engineering Decision Making

What are we up against?

Fukushima Nuclear Event
March 11, 2011

No fatalities ..?

Eco. imp. > 75 billion $US

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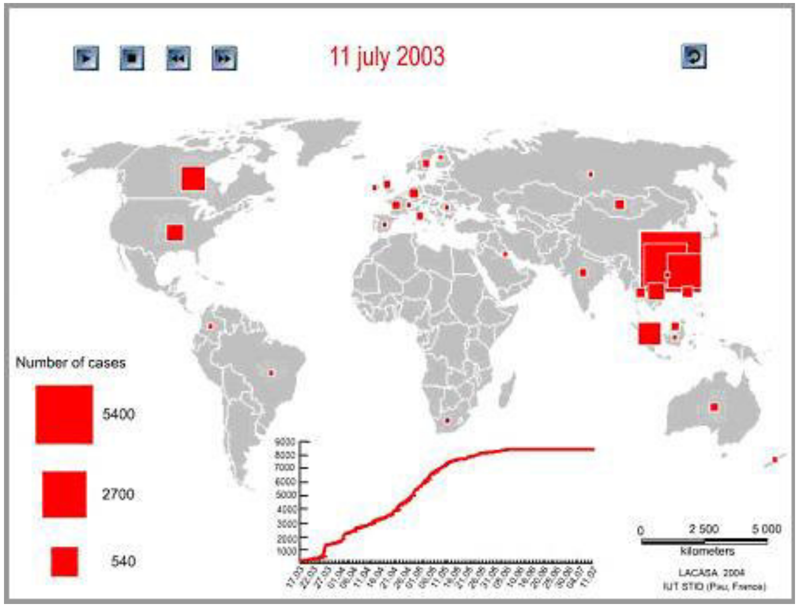
Context of Engineering Decision Making

What are we up against?



SARS, 2003
Fatalities: < 800
Eco. imp. 2% GDP – 200 billion $US

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Context of Engineering Decision Making

What are we up against?

Food borne diseases - USA
Affects 76 million per year
Hospitalizations: 325000 per year
Fatalities: 5000 pr year

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Decisions and Preferences

Attributes of decision outcomes

Decisions aim to achieve an objective

The degree of achievement is measured by attributes

- natural attributes (measurable, e.g. costs and loss of lives)
- constructed attributes (a function of natural attributes e.g. GDP)
- proxy attributes (indicators which measure the perceived degree of fulfilment of an objective)

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Decisions and Preferences

Preferences among attributes - utility

The attributes associated with a decision outcome may be translated into a degree of achievement of the objective by means of a utility function

different attributes are brought together on one or several scales

multi attribute decision making implies a weighing of different attributes

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Decisions and Preferences

Constraints on decision making

In principle – any society may define what they consider to be acceptable decisions

Typically decisions are constrained – e.g. in terms of maximum acceptable risks to

- persons
- qualities of the environment

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Uncertainty

Different types of uncertainties influence decision making

Inherent natural variability – aleatory uncertainty
- result of throwing dices
- variations in material properties
- variations of wind loads
- variations in rain fall

Model uncertainty – epistemic uncertainty
- lack of knowledge (future developments)
- inadequate/imprecise models (simplistic physical modelling)

Statistical uncertainties – epistemic uncertainty
- sparse information/small number of data

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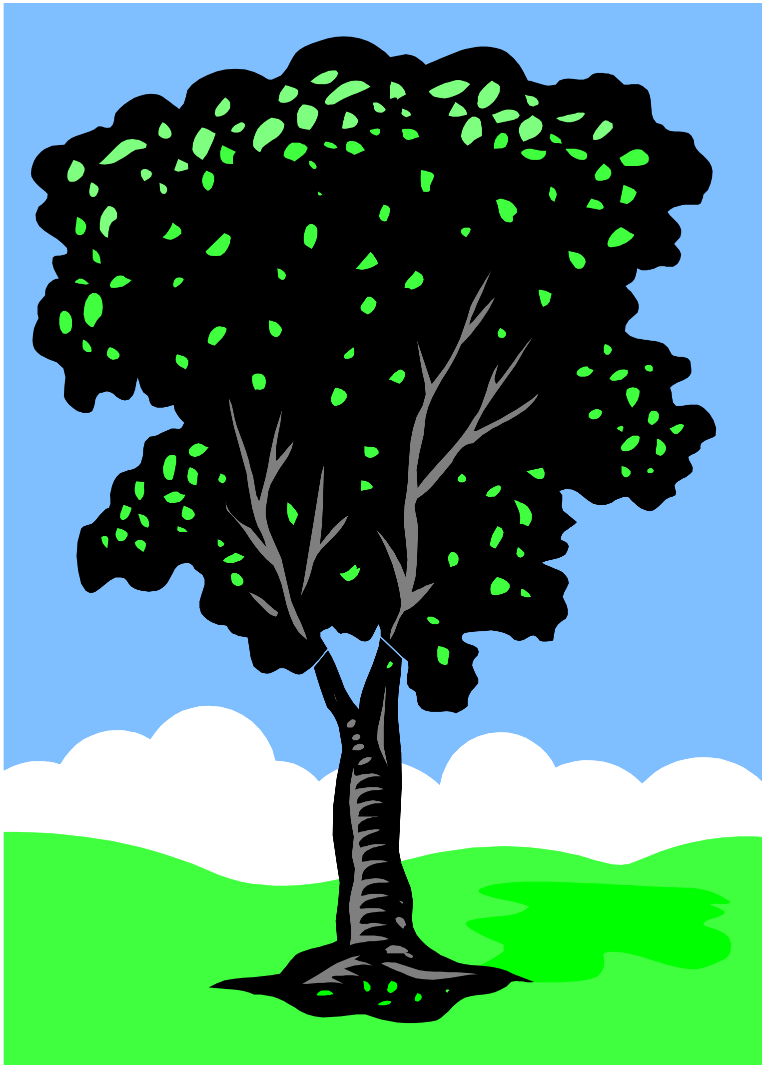
Probability



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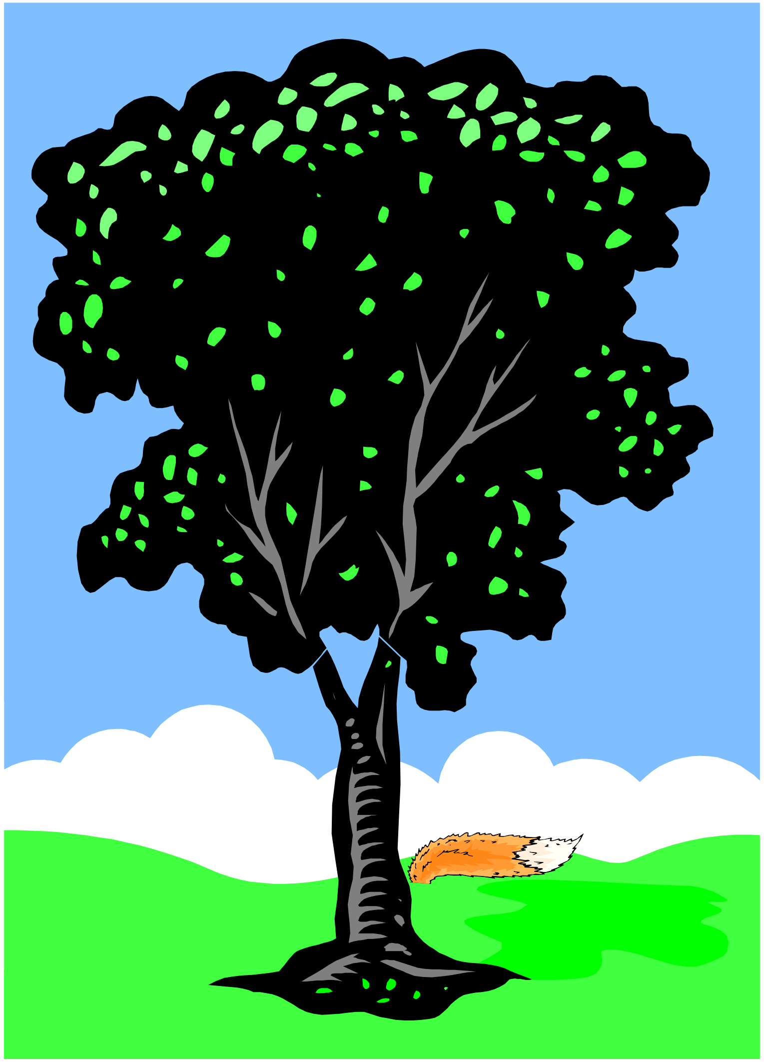
Probability



Probability

Probability

Formulate hypothesis about the world

Utilize existing knowledge

Combine with data

Learn how to develop knowledge !

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Probability

Conditional probabilities are of special interest as they provide the basis for utilizing new information in decision making.

The conditional probability of an event E_1 given that event E_2 has occurred is written as:

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Not defined if $P(E_2) = 0$

The event E_1 is said to be probabilistically independent of the event E_2 if:

$$P(E_1 | E_2) = P(E_1)$$

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Probability

From $$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

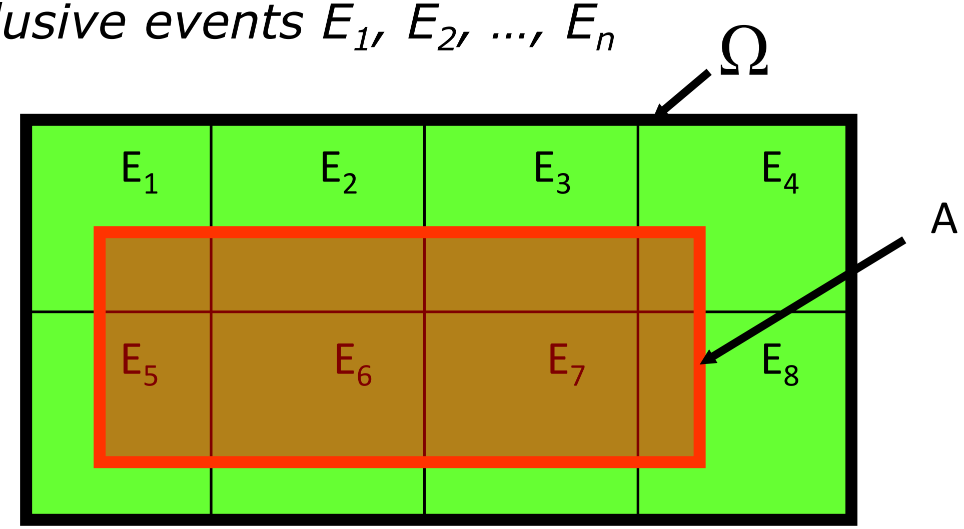
it follows that $$P(E_1 \cap E_2) = P(E_2) P(E_1 | E_2)$$

and when E_1 and E_2 are statistically independent it is

$$P(E_1 \cap E_2) = P(E_2) P(E_1)$$

Probability

Consider the sample space Ω divided up into n mutually exclusive events $E_1, E_2, \ldots, E_n$



$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_n)$$

$$P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \ldots + P(A|E_n)P(E_n) =$$

$$\sum_{i=1}^{n} P(A|E_i)P(E_i)$$

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Probability

as there is $P(A \cap E_i) = P(A|E_i)P(E_i) = P(E_i|A)P(A)$

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

Posterior

Bayes' Rule

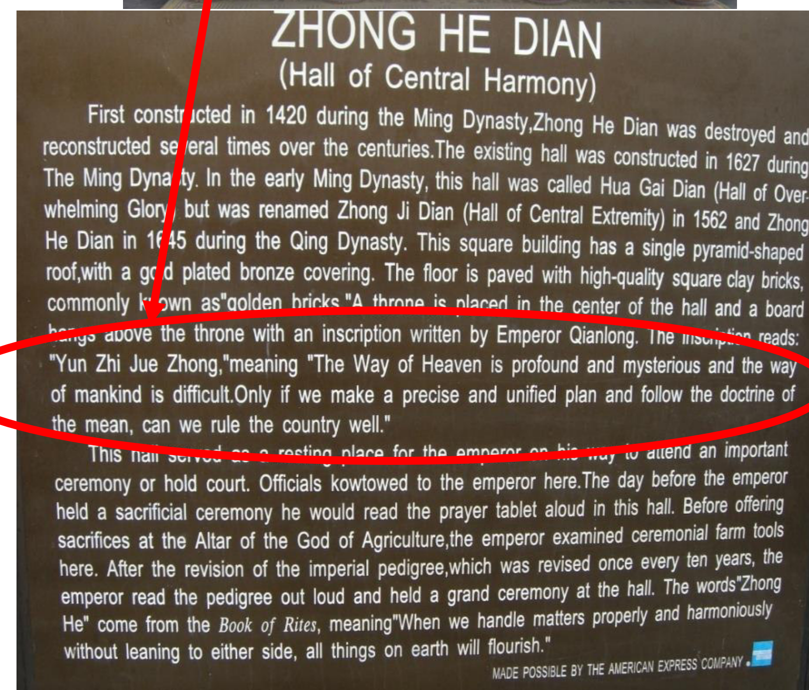
Reverend Thomas Bayes (1702-1764)

Decision Ranking

Emperor Qianlong
Qing dynasty
Reign :1735 – 1796

Daniel Bernoulli 1738
Expected utility hypothesis

von Neumann and Morgenstern 1947
4 Axioms of utility theory:
Ranking based on expected value
of utility (VNM rational)



Decision Ranking

Risk is a characteristic of an activity relating to all possible events n_E which may follow as a result of the activity

The risk contribution R_{E_i} from the event E_i is defined through the product between

the event probability P_{E_i}

and

the consequences of the event C_{E_i}

The risk associated with a given activity R_A may then be written as

$$R_A = \sum_{i=1}^{n_E} R_{E_i} = \sum_{i=1}^{n_E} P_{E_i} \cdot C_{E_i}$$

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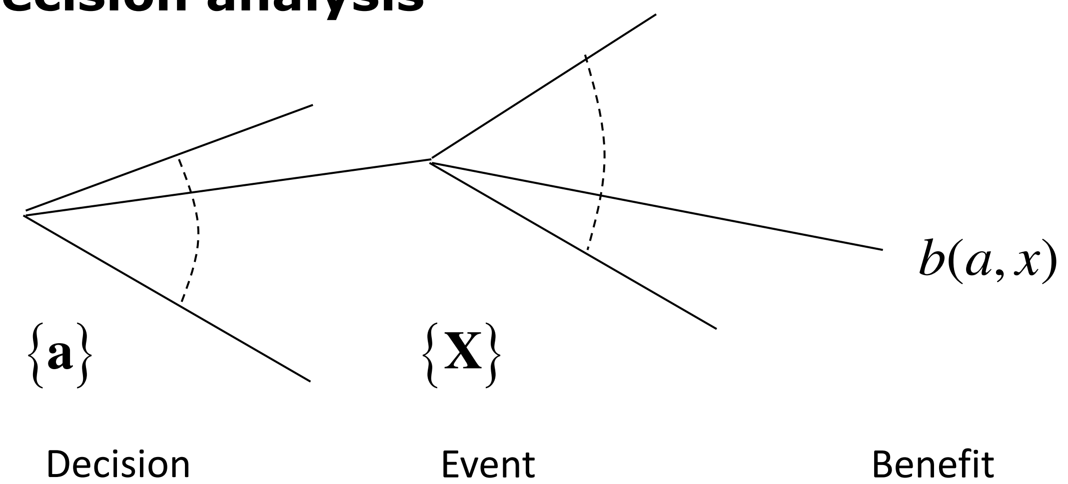
Decision Ranking

Prior decision analysis



Information is bought by choice of prior density

Optimal decision maximizes the expected value of utility (benefit) (von Neumann & Morgenstern)

$$B_0^* = \max_a E'\left[b(a,X)\right] = \max_a \int b(a,x) f_X'(x,a)dx$$

Decision Ranking

Posterior decision analysis

By sampling information $\mathbf{z}$ using an experiment e we may update the probabilistic description of X

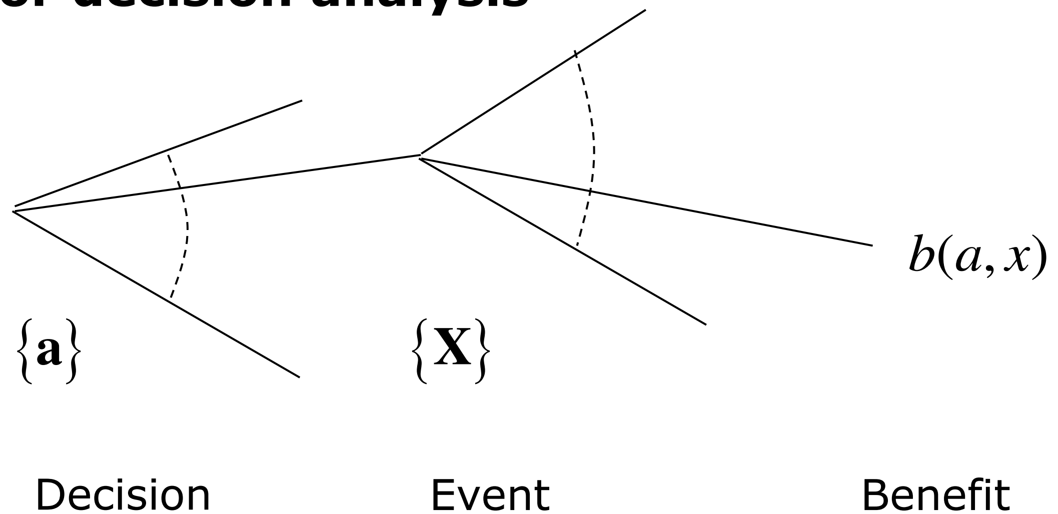
$$f_X''(x,a|\mathbf{z}) = \frac{L(x|\mathbf{z}) f_X'(x,a)}{\int L(x|\mathbf{z}) f_X'(x,a)}$$

Of course the likelihood of the sample $\mathbf{z}$ depends on the experiment e why we write

$$L(x|\mathbf{z}) = L(x|\mathbf{z}, e)$$

Decision Ranking

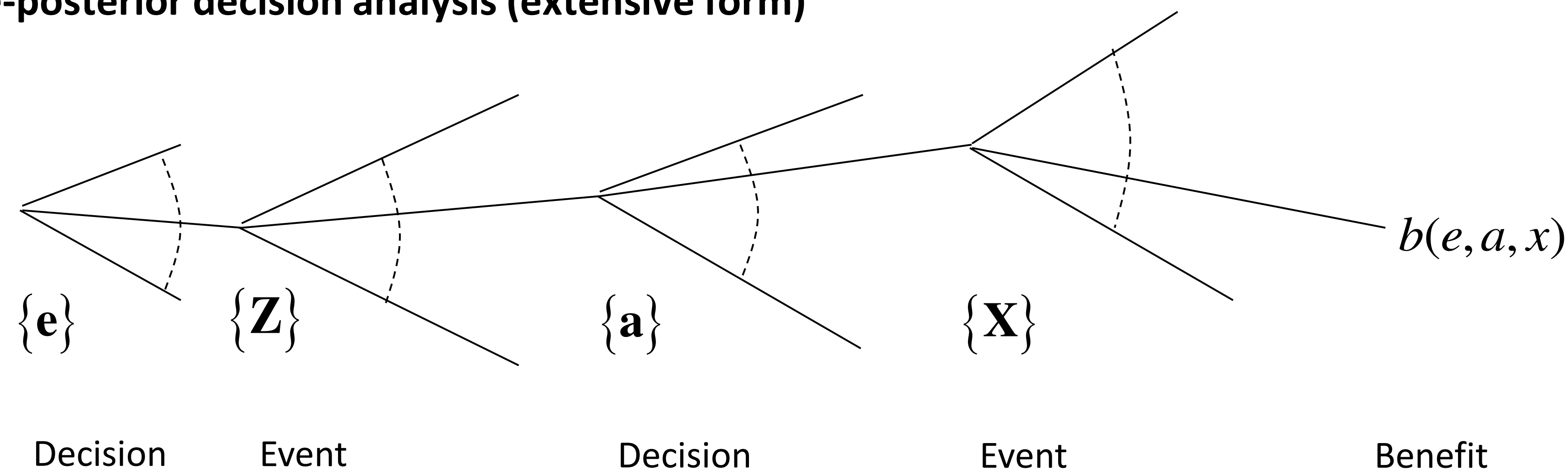
Posterior decision analysis



$$\max_a E''\left[b(a,X)\right] = \max_a \int b(a,x) f''_X(x,a|\hat{\mathbf{z}})dx$$

Decision Ranking

Pre-posterior decision analysis (extensive form)

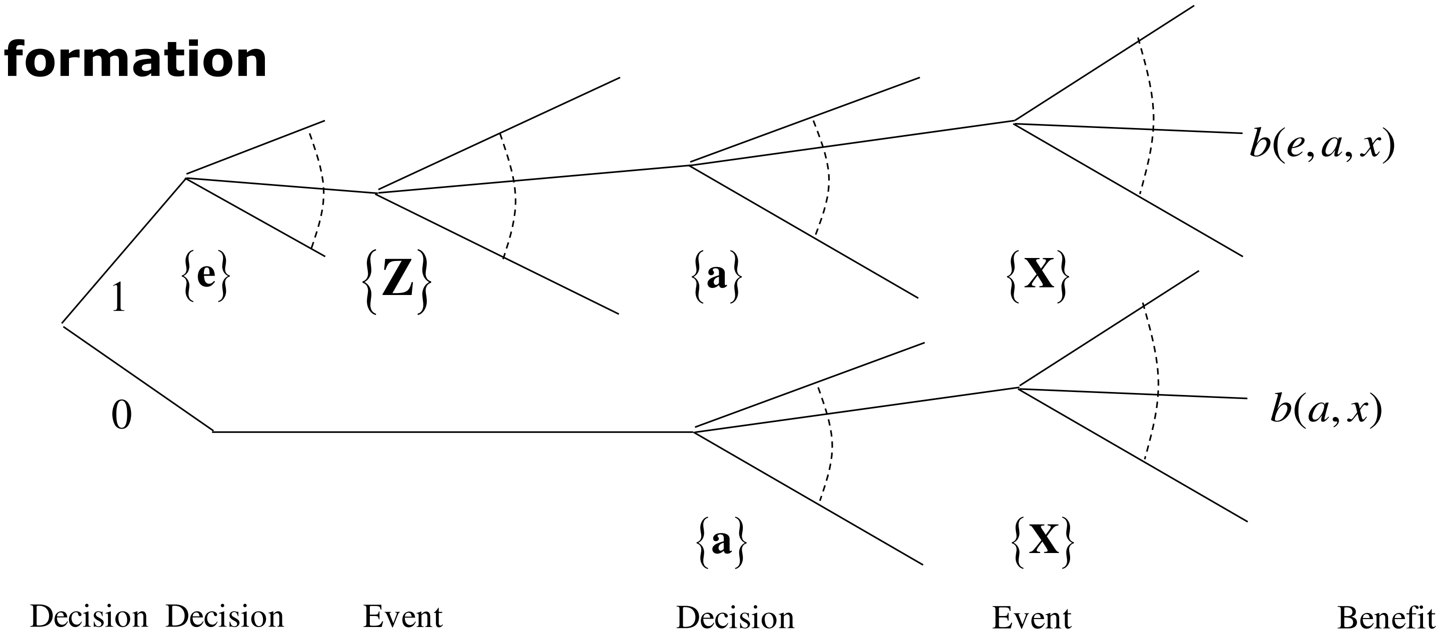


The optimal experiment e may be found from

$$B_1^* = \max_e E_{\mathbf{Z}}\left[\max_a \int b(e,a,x) f_X''(x,a|\mathbf{Z})dx\right]$$

Decision Ranking

Value of Information



The value of information *VoI* is determined from:

$$VoI = \max_e E_{\mathbf{Z}}\left[\max_a \int b(e,a,x) f_X''(x,a|\mathbf{Z})dx\right] - \max_a \int b(a,x) f_X'(x,a)dx$$

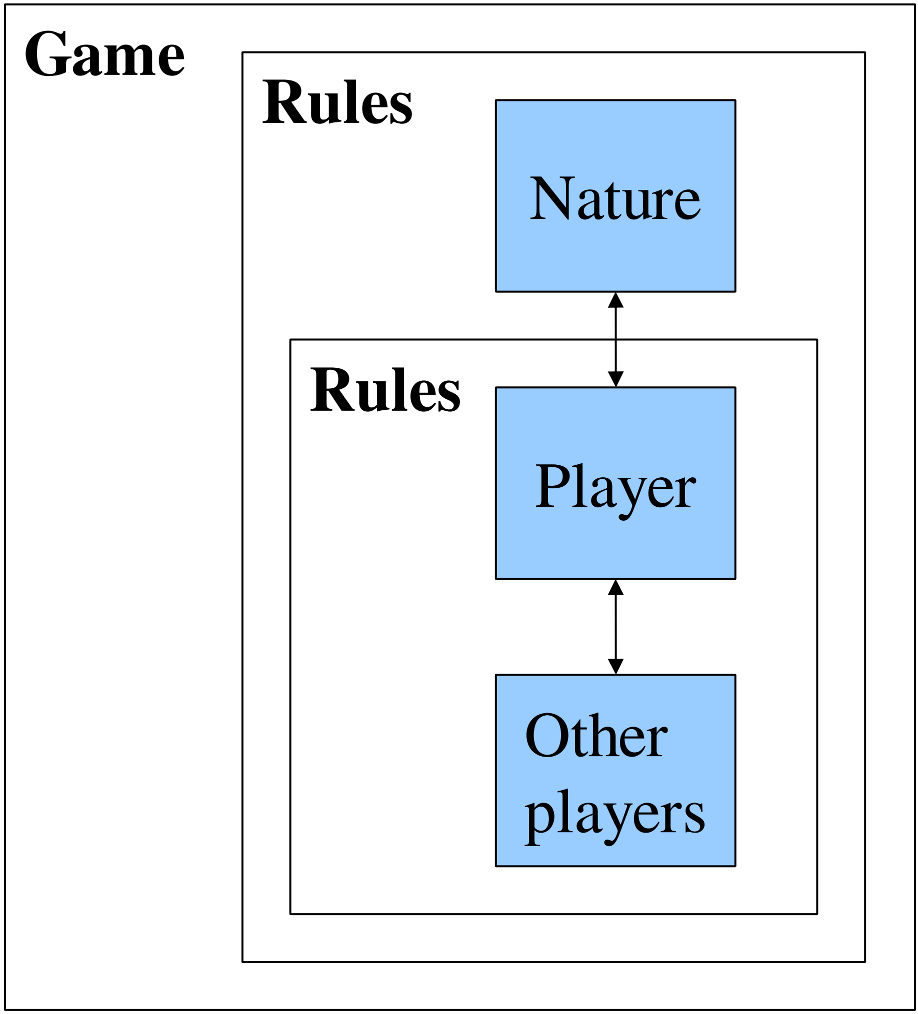
Decision Ranking

Games and Risk

Rules (exogenous)
- Nature

Rules (endogenous)
- Knowledge
- Best practices
- Rules and standards
- Culture
- Ethics

Drivers/Challenges
- Preferences
- Psychology
- Asymmetric information



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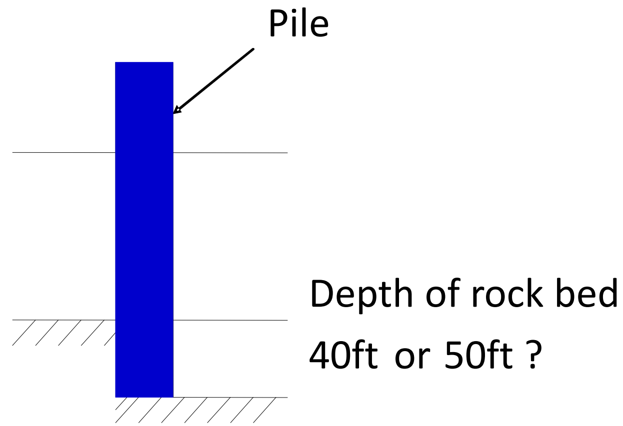
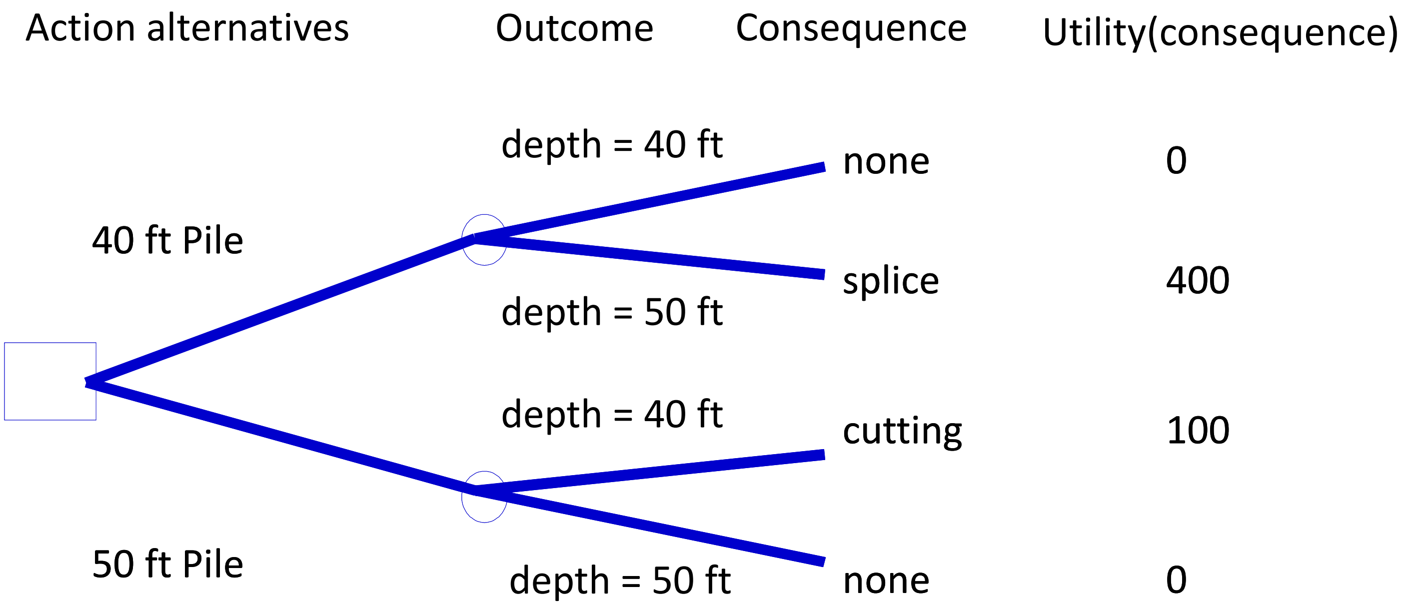
Decision Analysis in Engineering

The decision tree





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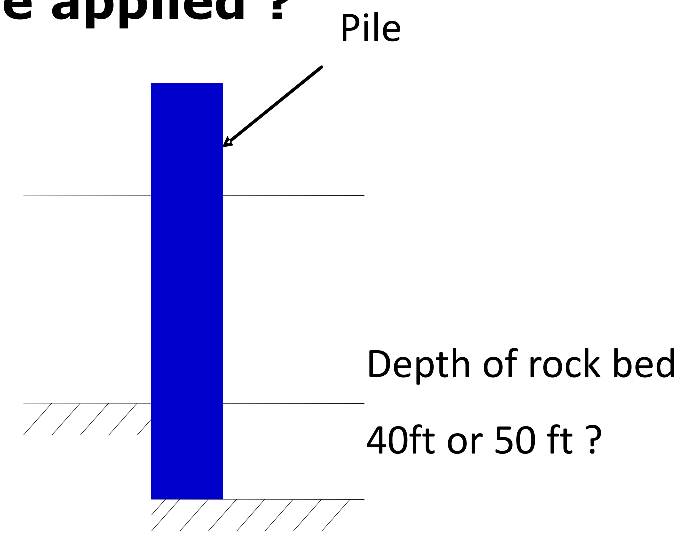
Decision Analysis in Engineering

The different types of decision analysis

- **Prior**
- **Posterior**
- **Pre-posterior**

Illustrated on an example :

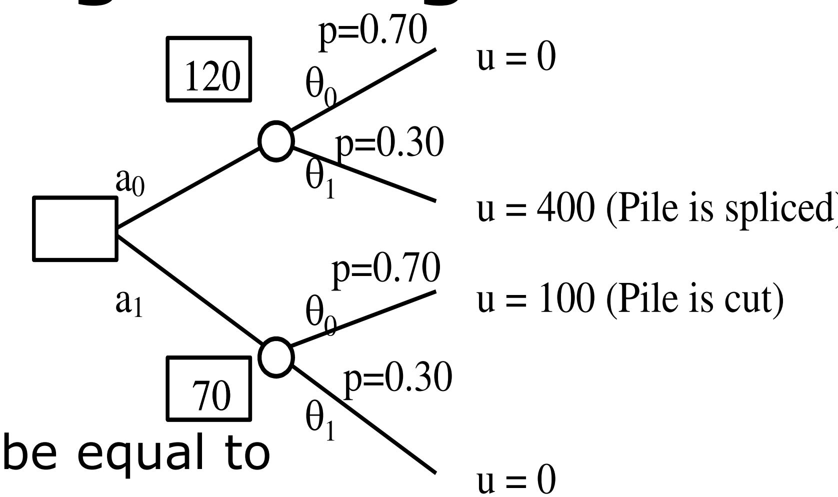
Question : What pile length should be applied ?



Alternatives :
a_0 : Choose a 40 ft pile
a_1 : Choose a 50 ft pile

States of nature (depth to rock bed)
θ_0 : Rock bed at 40 ft
θ_1 : Rock bed at 50 ft

Decision Analysis in Engineering



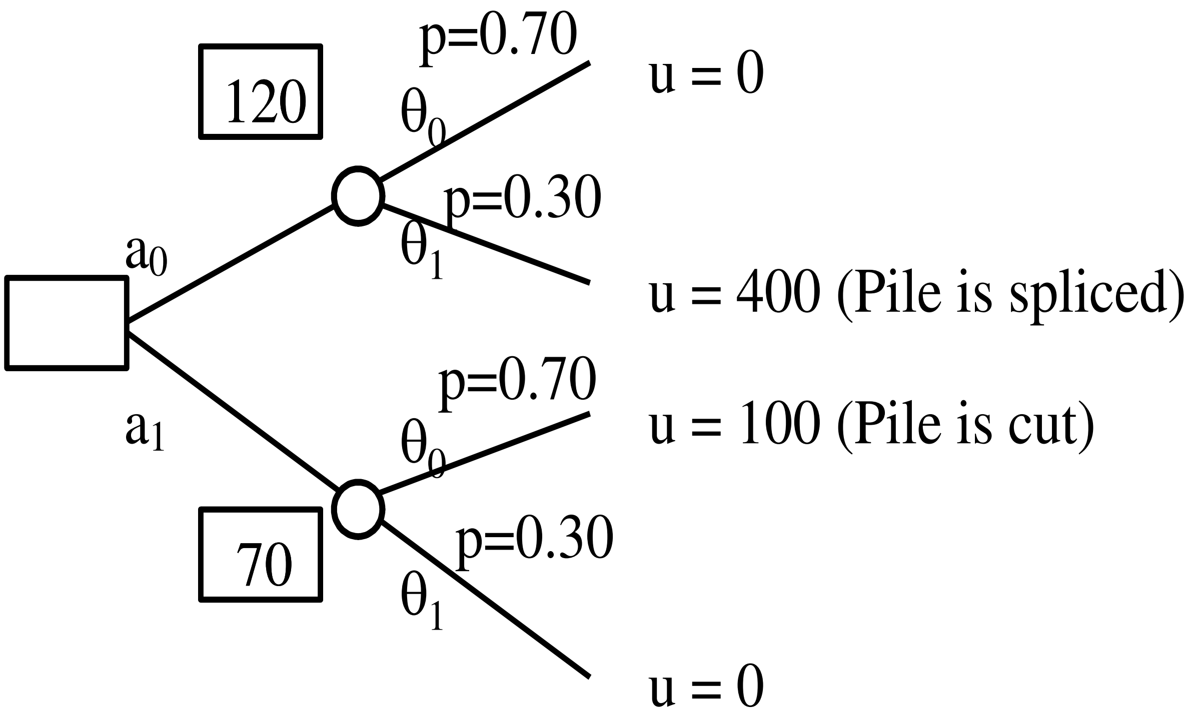
Prior Analysis

$P'[q_0] = 0.70$
$P'[q_1\ ] = 0.30$

The expected utility is calculated to be equal to

$$
\begin{aligned}
E'[u] &= \min\{u[a_0]\ , u[a_1]\} \\
&= \min\{P'[\theta_0] \times u[\theta_0 | a_0] + P'[\theta_1] \times u[\theta_1 | a_0]\ , \\
&\qquad P'[\theta_0] \times u[\theta_0 | a_1] + P'[\theta_1] \times u[\theta_1 | a_1]\} \\
&= \min\{0.7 \times 0 + 0.3 \times 400,\ 0.7 \times 100 + 0.3 \times 0\} \\
&= \min\{120, 70\} = 70 \quad \Rightarrow \text{Decision for } \mathrm{a}_1 \text{ (50ft Pile)}
\end{aligned}
$$

Decision Analysis in Engineering



$\Longrightarrow$ Choice of pile a_1 (50ft Pile)

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Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \,|\theta_i\right]P'\left[\theta_i\right]}{\sum_j P\left[z_k \,|\theta_j\right]P'\left[\theta_j\right]}$$

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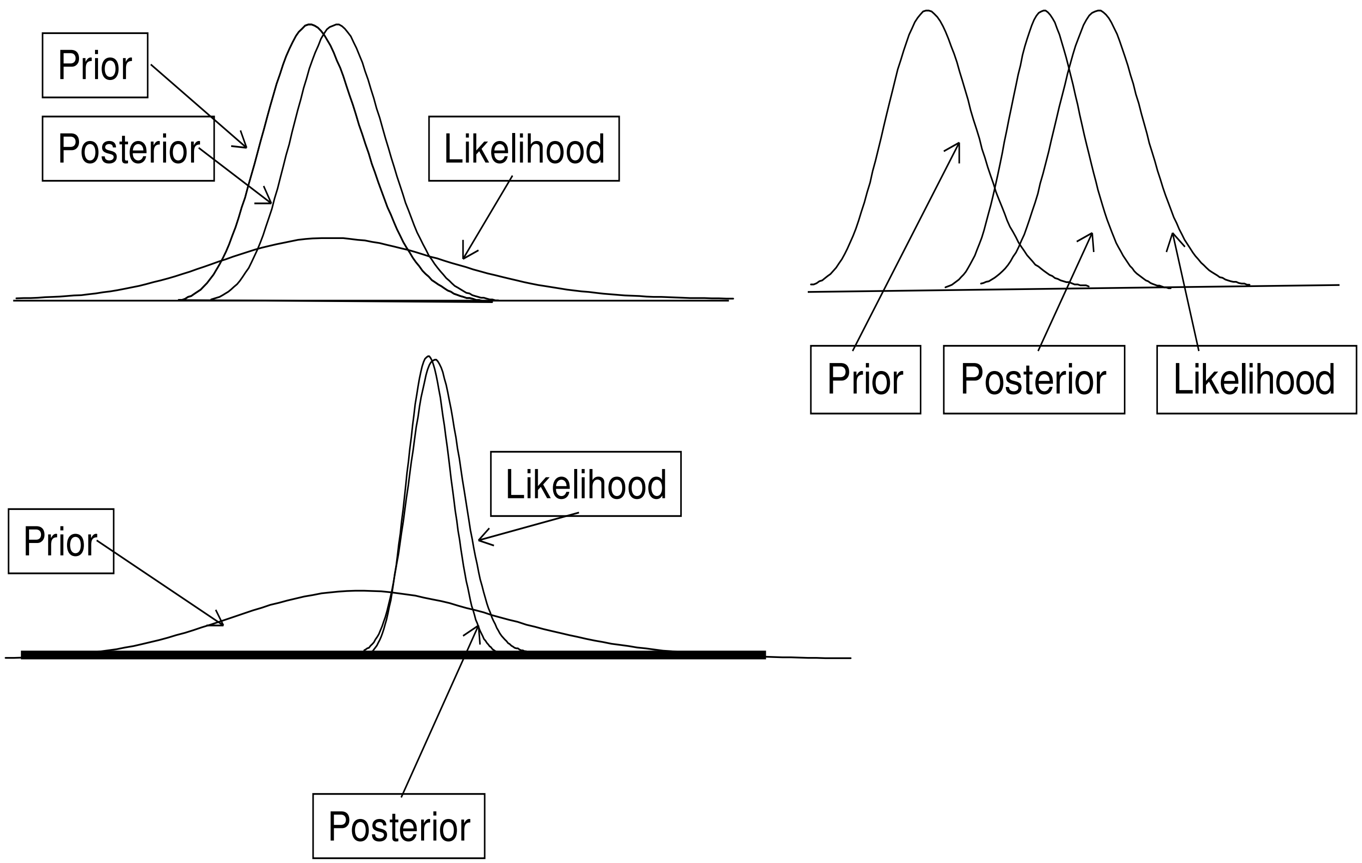
Decision Analysis in Engineering



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Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

Ultrasonic tests to determine the depth to bed rock

True state / Test result	θ_0 40 ft – depth	θ_1 50 ft – depth
z_0 - 40 ft indicated	0.6	0.1
z_1 - 50 ft indicated	0.1	0.7
z_2 - 45 ft indicated	0.3	0.2

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P\left[z_k \middle| \theta_j\right]$

Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

It is assumed that a test gives a 45 ft indication

$$P''\left[\theta_0\right] = P\left[\theta_0 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_0\right] P\left[\theta_0\right] = 0.3\ x\ 0.7 = 0.21$$

$$P''\left[\theta_1\right] = P\left[\theta_1 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_1\right] P\left[\theta_1\right] = 0.2\ x\ 0.3 = 0.06$$

$$P''\left[\theta_0 \middle| z_2\right] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''\left[\theta_1 \middle| z_2\right] = \frac{0.06}{0.21 + 0.06} = 0.22$$

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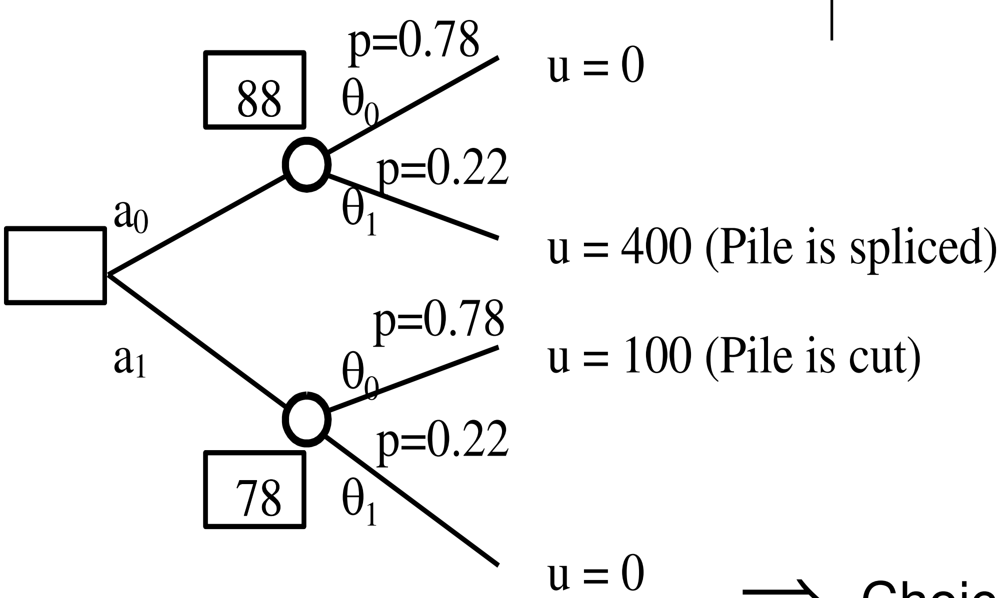
Posterior Analysis

Test result indicates 45ft to rock bed



$\Longrightarrow$ Choice of alternative a_1 (50ft Pile)

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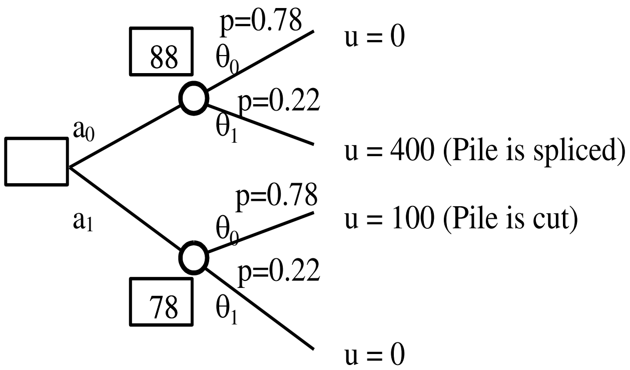
Decision Analysis in Engineering

Posterior Analysis



$$E''\left[u \middle| z_2\right] = \min_j\{E''\left[u(a_j) \middle| z_2\right]\}$$

$$= \min\{P''[\theta_0]\times 0 + P''[\theta_1]\times 400,\ P''[\theta_0]\times 100 + P''[\theta_1]\times 0\}$$

$$= \min\{0.78\times 0 + 0.22\times 400,\ 0.78\times 100 + 0.22\times 0\}$$

$$= \min\{88\ ,\ 78\} = 78$$

$\Longrightarrow$ Choice of alternative a_1 (50ft Pile)

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Decision Analysis in Engineering

Pre-posterior Analysis

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''\left[u \middle| z_i\right] = \sum_{i=1}^{n} P'[z_i] \times \min_{j=1,m}\{E''\left[u(a_j) \middle| z_i\right]\}$$

$$P'[z_i] = P\left[z_i \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_i \middle| \theta_1\right] \times P'[\theta_1]$$

$$P'[z_0] = P\left[z_0 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_0 \middle| \theta_1\right] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

$$P'[z_1] = P\left[z_1 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_1 \middle| \theta_1\right] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P\left[z_2 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_2 \middle| \theta_1\right] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

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Decision Analysis in Engineering

Pre-posterior Analysis

$$E''\left[u\middle|z_0\right]=\min_j\{E''\left[u(a_j)\middle|z_0\right]\}$$

$$\overbrace{\textbf{do nothing} \qquad \textbf{splicing}}^{\mathbf{a_0}} \qquad \overbrace{\textbf{cutting} \qquad \textbf{do nothing}}^{\mathbf{a_1}}$$

$$=\min\{P''\left[\theta_0\middle|z_0\right]\times 0+P''\left[\theta_1\middle|z_0\right]\times 400,\ P''\left[\theta_0\middle|z_0\right]\times 100+P''\left[\theta_1\middle|z_0\right]\times 0\}$$

$$=\min\{0.93\times 0+0.07\times 400,\ 0.93\times 100+0.07\times 0\}$$

$$=0.07\times 400+0.93\times 0=28$$

Decision Analysis in Engineering

Pre-posterior Analysis

$$E''\left[u \middle| z_1\right] = \min_j \{E''\left[u(a_j) \middle| z_1\right]\}$$

$$= \min\{\underbrace{\underbrace{P''\left[\theta_0 \middle| z_1\right]\times 0}_{\text{do nothing}} + \underbrace{P''\left[\theta_1 \middle| z_1\right]\times 400}_{\text{splicing}}}_{a_0},\ \underbrace{\underbrace{P''\left[\theta_0 \middle| z_1\right]\times 100}_{\text{cutting}} + \underbrace{P''\left[\theta_1 \middle| z_1\right]\times 0}_{\text{do nothing}}}_{a_1}\}$$

$$= \min\{0.25\times 0 + 0.75\times 400,\ 0.25\times 100 + 0.75\times 0\}$$

$$= 0.25\times 100 + 0.75\times 0 = 25$$

Decision Analysis in Engineering

Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis
– not including costs of experiments

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.00$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$

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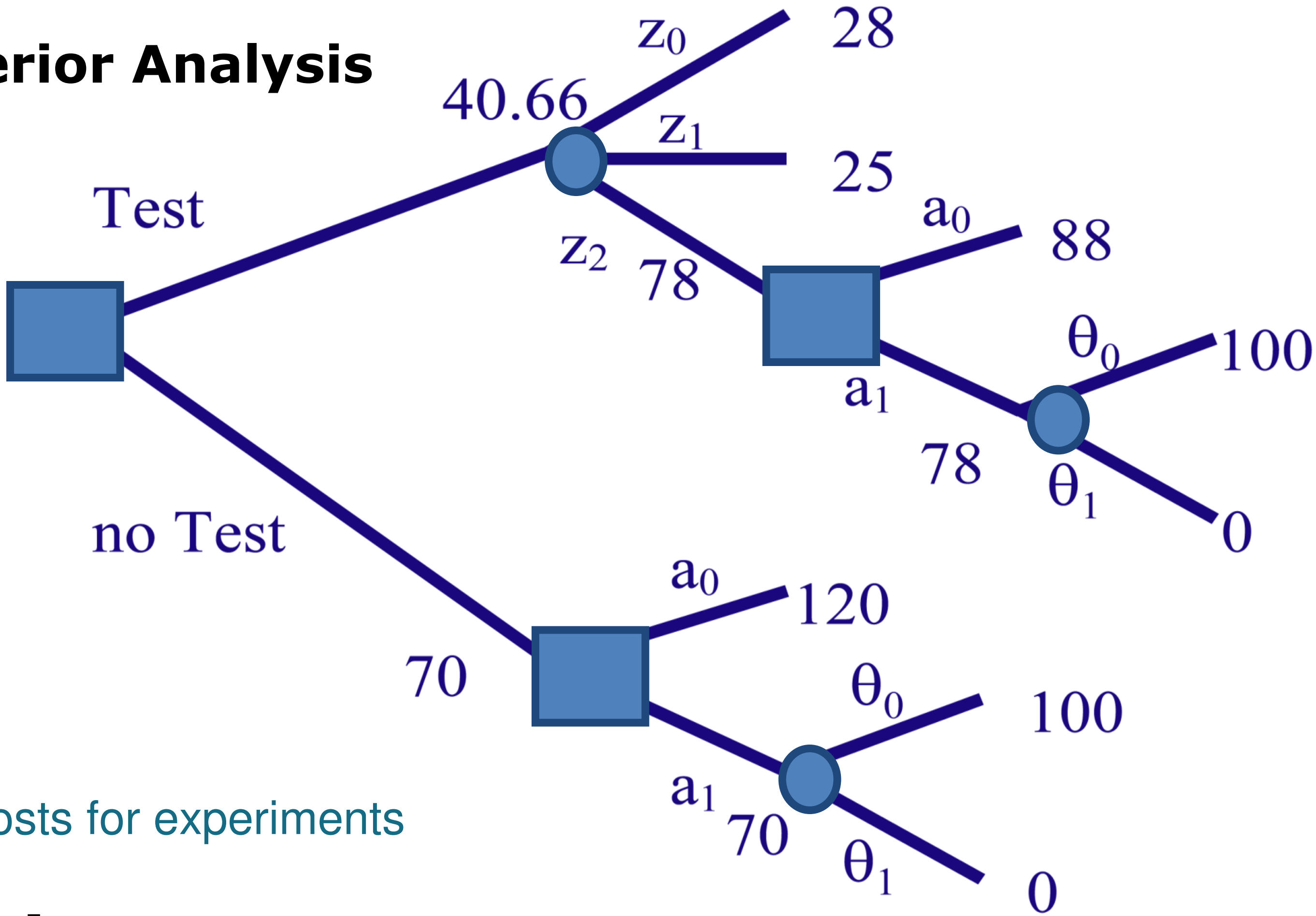
Decision Analysis in Engineering

Pre-posterior Analysis



Allowable costs for experiments

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$

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